

# STATISTICAL SIGNAL

## ○ PROCESSING

This course is concerned with the processing of signals in the presence of uncertainty, using the framework of probability and statistics. Let's look at these terms in more detail.

### Signals, Processing, and Uncertainty

#### Signals

○ In this course, a signal is any set of measurements that can be represented by a list of real or complex numbers

$$x_1, x_2, \dots, x_N$$

We will think of a signal as a Euclidean vector

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \in \mathbb{R}^N \text{ or } \mathbb{C}^N$$

Examples |

○

Ⓐ

Processing

Analyze, modify, synthesize

○

Examples |

Ⓑ

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## Uncertainty

- Noise / randomness, unknowns

### Examples

○ (c)

- sensor noise

- variability inherent in nature

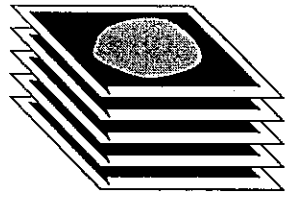
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- environmental effects

- unknown signal parameters

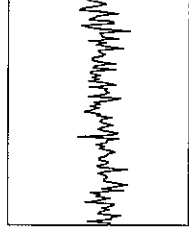
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# Functional Magnetic Resonance Imaging

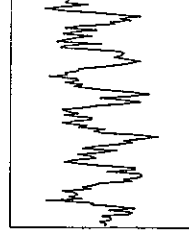


time

brain image  
time-series



“inactive” pixel  
time-series



“active” pixel  
time-series



statistical signal processing



activation map

## Challenges:

- measurement noise
- intrinsic uncertainties in signal behavior

# Probability and Statistics

## ○ Probability Theory

Probability theory is the most widely accepted and commonly used approach to modeling uncertainty, although alternatives exist, such as fuzzy logic.

Probability theory models uncertainty by specifying

- the chance of observing certain signals
- the degree to which we believe a signal reflects the true state of nature.

## ○ Examples of probabilistic models

- sensor noise modeled as an additive Gaussian random variable

① uncertainty in the phase of a sinusoidal signal modeled as a \_\_\_\_\_ random variable on  $[0, 2\pi)$ .

- uncertainty in the number of photons striking a CCD per unit time modeled as a \_\_\_\_\_ random variable.

## Statistical Inference

A statistic is a function of observed data, and may be scalar or vector valued.

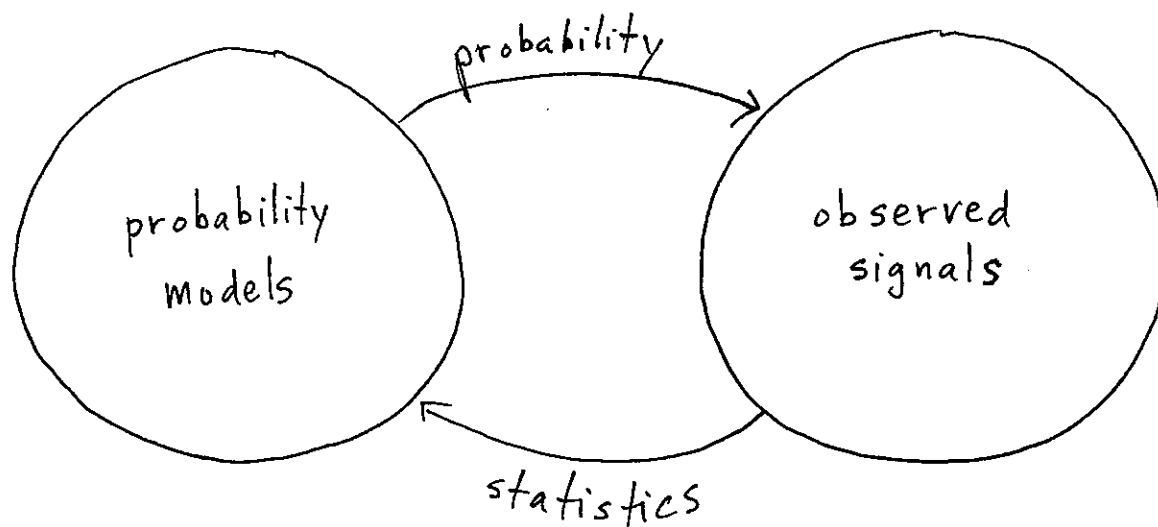
Examples | Suppose we observe  $N$  scalar values  $x_1, \dots, x_N$ . The following are statistics

- sample mean  $\rightarrow \bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$
- the data itself  $\rightarrow [x_1, \dots, x_N]^T$
- an order statistic  $\rightarrow x_{(1)} = \min\{x_1, \dots, x_N\}$
- an arbitrary function  $\rightarrow [x_1^2 - x_2 \sin(x_3), e^{-x_1 x_3}]^T$

A statistic cannot depend on unknown quantities.

Probability laws describe the uncertainty in the signals we might observe.

Statistics describe the salient features of the signals we do observe, and allow us to draw conclusions (inferences) about which probability model actually reflects the true state of nature



# Statistical Signal Processing (in a nutshell)

Step 1: Postulate a probability model (or models) that can be expected to reasonably capture the uncertainties in the data.

Step 2: Collect data

Step 3: Formulate statistics that allow us to interpret or understand our probability models.

In this class we will focus on three areas:

- Estimation
- Filtering
- Detection



Hence the name



## Estimation

- Estimate an unknown signal parameter by considering a family of probability models indexed by that parameter and selecting the model that best captures the data.

### Examples 1

Consider a signal in additive noise

$$x(n) = s(n) + w(n), \quad n=0, 1, \dots, N-1$$

- • Sinusoidal parameter estimation

$$s(n) = A \cos(2\pi f n + \phi)$$

$A, f, \phi$  unknown

- Delay estimation

$$s(n) = r(n-d)$$

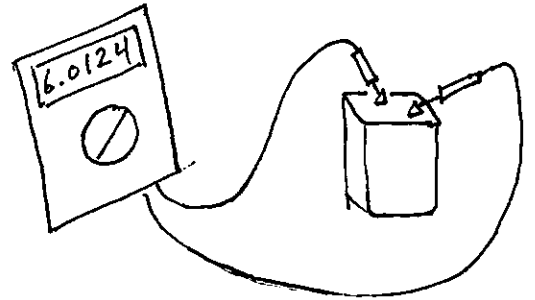
where  $r(n)$  is a known signal but  $d$  is unknown.

○

# A more concrete estimation example.

Example | Suppose we measure the voltage

$A$  of a battery using a voltmeter. Because the voltmeter tends to pick up noise from nearby objects, we take  $n$  measurements in hopes of gaining some accuracy.



Step 1: Assume a Gaussian noise model

$$x_i = A + w_i, \quad i=1, \dots, N$$

where

$$w_i \sim N(0, \sigma_w^2)$$

Step 2: Gather data

Step 3: Estimate  $A$  via the sample mean

$$\hat{A} = \bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

Is this the "right" statistic for this noise model? How accurate is the estimate? What if  $\sigma_w^2$  is unknown?

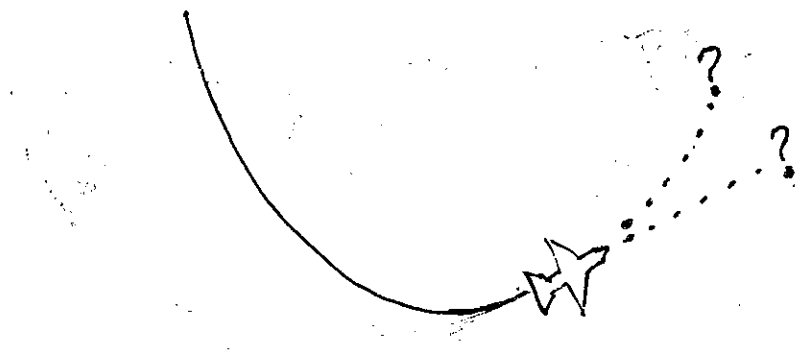
## Filtering

Filtering in deterministic SP refers to modifying a signal by means of a linear operator (usually expressed in terms of convolution).

In statistical SP, filtering refers to signal estimation by means of a linear function of the data. Thus filtering is a special case of estimation, but it is distinguished by being concerned with online/real-time estimation of streaming data.

## Examples

- Denoise a speech / audio signal real-time
- Track a moving target or predict its future location



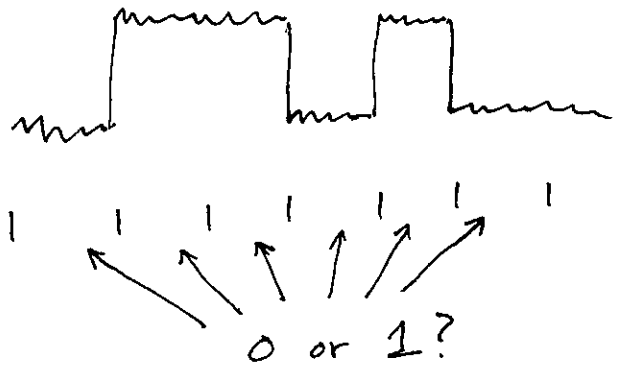
## Detection

Given two (or more) probability models, which one best explains the observed signal?

Alternatively, given a single probability model, is it or is it not a valid characterization of the data?

## Examples

- Decode a comm. signal into a sequence of 0's and 1's.
- Are other ships present on radar/sonar, and if so, where are they (detection and estimation)
- Is a supernova exploding somewhere in the field of view of a certain telescope?



A more concrete detection example.

Example 1 | Suppose you are given a coin and are asked to determine whether the following hypothesis is true:

$H_0$ : the coin is fair

Step 1: Assume each toss of the coin is a realization of a  Bernoulli  random var.

$$X \sim \text{Bernoulli}(p)$$

where  $p = \text{Prob}\{\text{heads}\}$ .

Step 2: Toss the coin  $n = 100$  times

$$x_i = 1 \iff \text{heads}$$

$$x_i = 0 \iff \text{tails}$$

Step 3: To assess the hypothesis

$$H_0: p = \frac{1}{2},$$

form the statistic

$$k = \sum_{i=1}^{100} x_i$$

and reject  $H_0$  if  $|k - 50| > 10$ .

In these examples, we used our intuition and heuristics to solve estimation and detection problems.

In this course we will develop principled and mathematically rigorous approaches to estimation, filtering, and detection using the theoretical framework of probability and statistics.

- Key**
- (A) sampled waveform, such as speech; digital image; Dow-Jones index closing values; packet counts along various links in a network; vector of medical attributes; ...
  - (B) speech: speaker identification, pitch alteration, speech synthesis; images: edge detection, noise removal, deblurring; ...
  - (C)
    - grainy images, old vinyl records ...
    - stock market, Internet ...
    - multipath signals in wireless comm, ambient EM waves, radar jamming
    - delay of radar return, pitch of speech signal
  - (D) uniform, Poisson
  - (E) Bernoulli,  $X \sim \text{Ber}(p)$