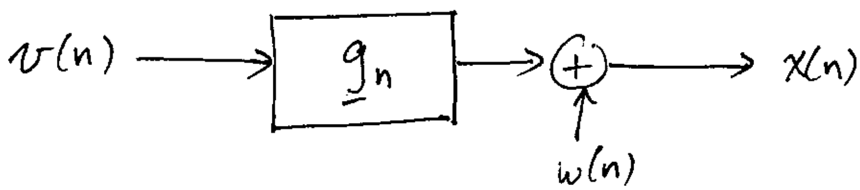
APPLICATION: CHANNEL ESTIMATION

Time Varying Channel Estimation

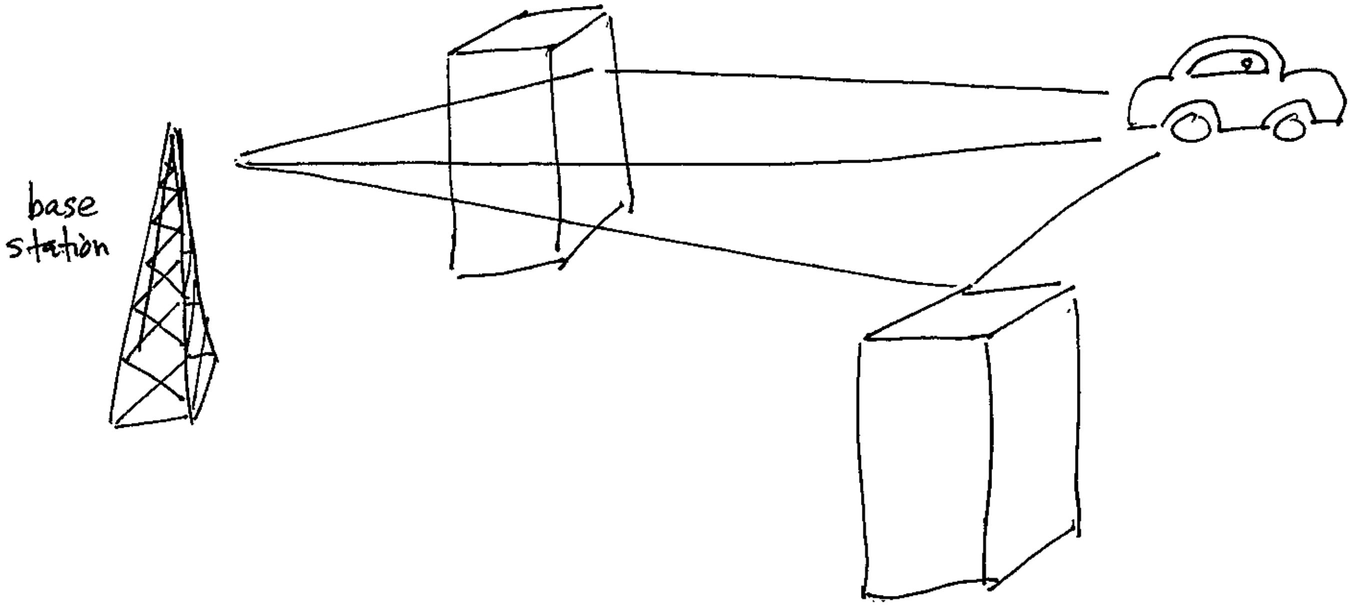
Recall the problem of time-varying channel estimation



$$x(n) = \sum_{k=0}^{p-1} g_n(k)\, v(n-k) + w(n)$$

Here $v(n)$ is known, $x(n)$ is observed, and the goal is to estimate the filter (channel) $\underline{g}_n$ which varies with time. The filter length p is assumed known.

Motivation: Wireless Communication



In wireless comm, multipath propogation channels cause the received signal to be a superposition of attenuated and delayed versions of the transmitted signal. This effect is called <u>fading</u>. It can be represented by an FIR filter which is called a <u>fading channel</u>.

Furthermore, if the transmitter, receiver, or environment are moving, the channel is time-varying.

For reliable communication, it is important to know the channel. For example, if the channel introduces a lot of distortion, it may be necessary to transmit at a greater power to maintain a given bit rate.

Now let's think about channel estimation. We observe

$$x(1) = \sum_{k=0}^{p-1} g_1(k)\, v(1-k) + w(1)$$

$$x(2) = \sum_{k=0}^{p-1} g_2(k)\, v(2-k) + w(2)$$

$$\vdots$$

Notice that there are p unknowns for each equation, so even without noise, the system of equations is highly underdetermined.

What we need is <u>prior information</u> that relates the different $\underline{g}_n$'s (channels).

In wireless comm, a common assumption is <u>slow fading</u>, which states that the channel changes slowly. For example, a car moves slowly relative to the sampling rate of a comm. system.

Probabilistically, we can express slow fading in terms of a vector-valued Gauss-Markov process:

$$\underline{g}_n = A\,\underline{g}_{n-1} + \underline{u}_n$$

where A is "close" to the identity and

$$\underline{u}_n \overset{iid}{\sim} N(\underline{0}, Q)$$

is driving noise with covariance Q.

$\boxed{\text{Kalman Filter for Channel Estimation}}$

With this slow-fading model we can now apply a Kalman filter

<u>State equation</u>

$$\underline{g}_n = A \cdot \underline{g}_n + \underline{u}_n$$

$$(\text{with } \underline{g}_n \text{ in place of } \underline{s}(n))$$

<u>Measurement equation</u>

$$x(n) = H \cdot \underline{g}_n + w(n)$$

where $$H = [v(n-p+1) \quad \dots \quad v(n)]$$

Notice that H depends on n. This is OK, our derivation of the Kalman filter equations did not assume H was static.

Example ($p = 2$)

Slow-Fading Model Parameters

$$A = \begin{bmatrix} 0.999 & 0 \\ 0 & 0.999 \end{bmatrix}$$ ← state transition matrix

$$Q = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix}$$ ← driving noise covariance

to reflect little or no knowledge about the initial state of the channel

$$\underline{g}_{-1} \sim N\left(\underline{0}, \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}\right).$$

Channel model:

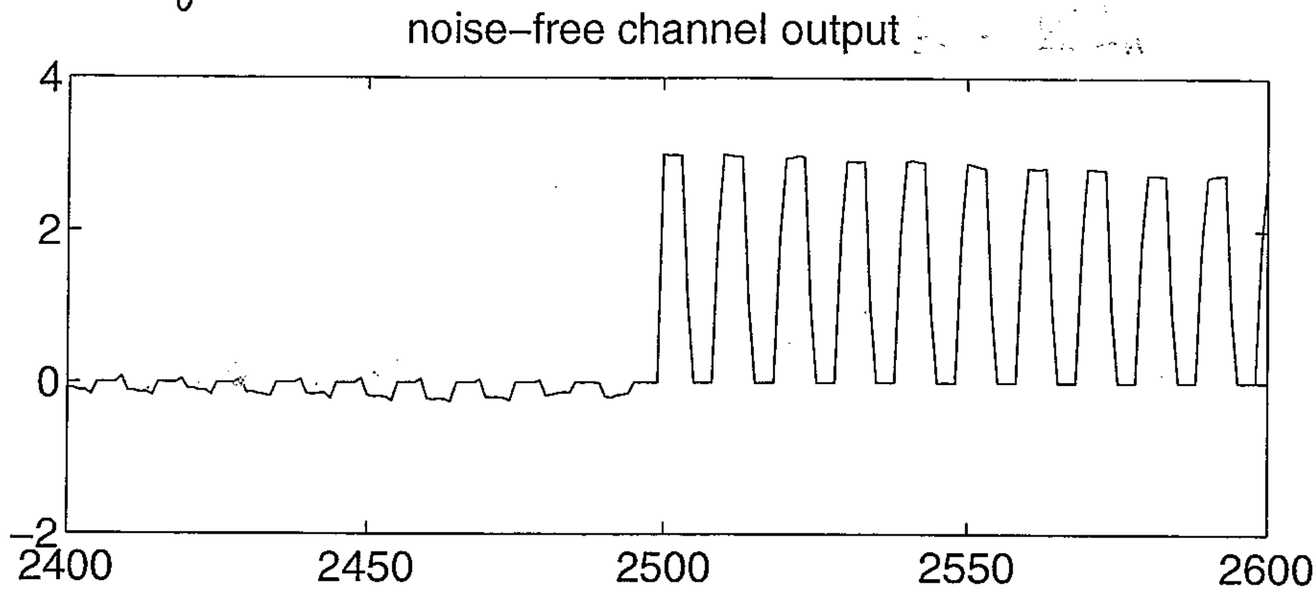
$$x(n) = g_n(0)\, v(n) + g_n(1)\, v(n-1) + w(n)$$

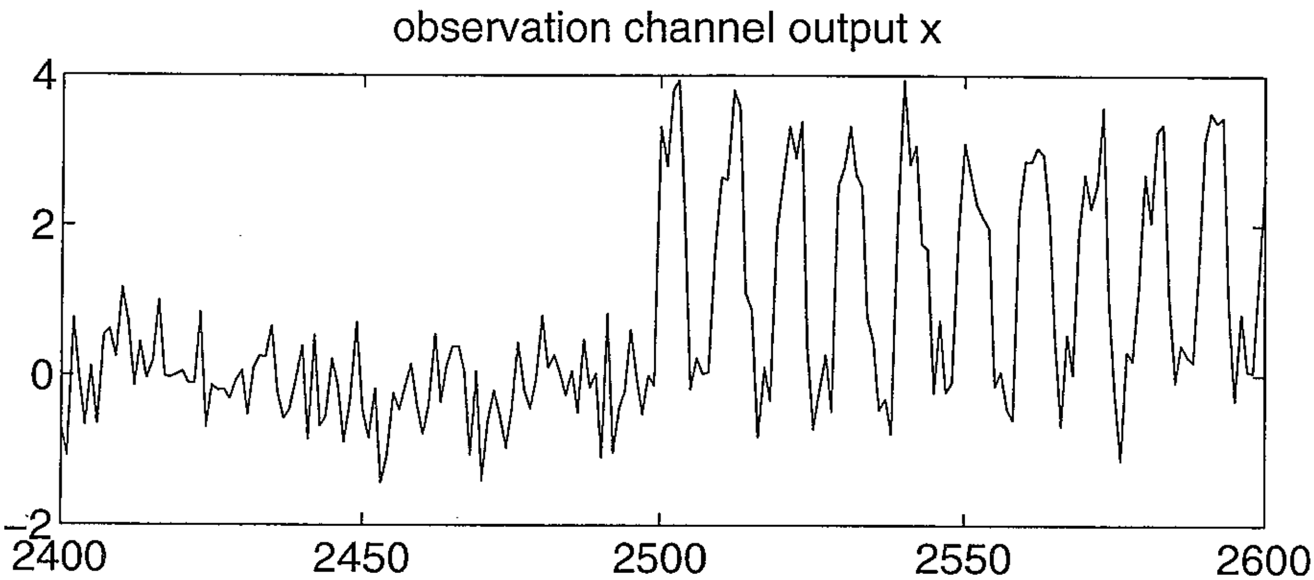
$v(n)$ = input to channel, known square wave

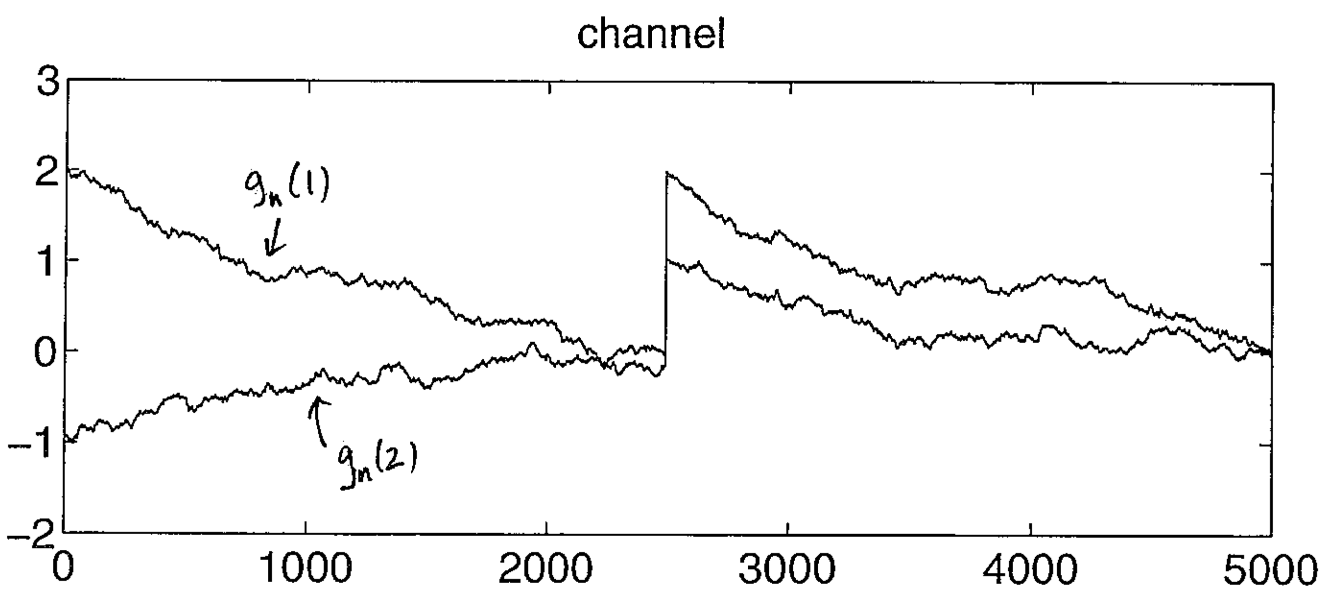
$g_n(k)$ = time-varying channel model (linear time-varying FIR filter)

$w(n)$ = white Gaussian observation noise

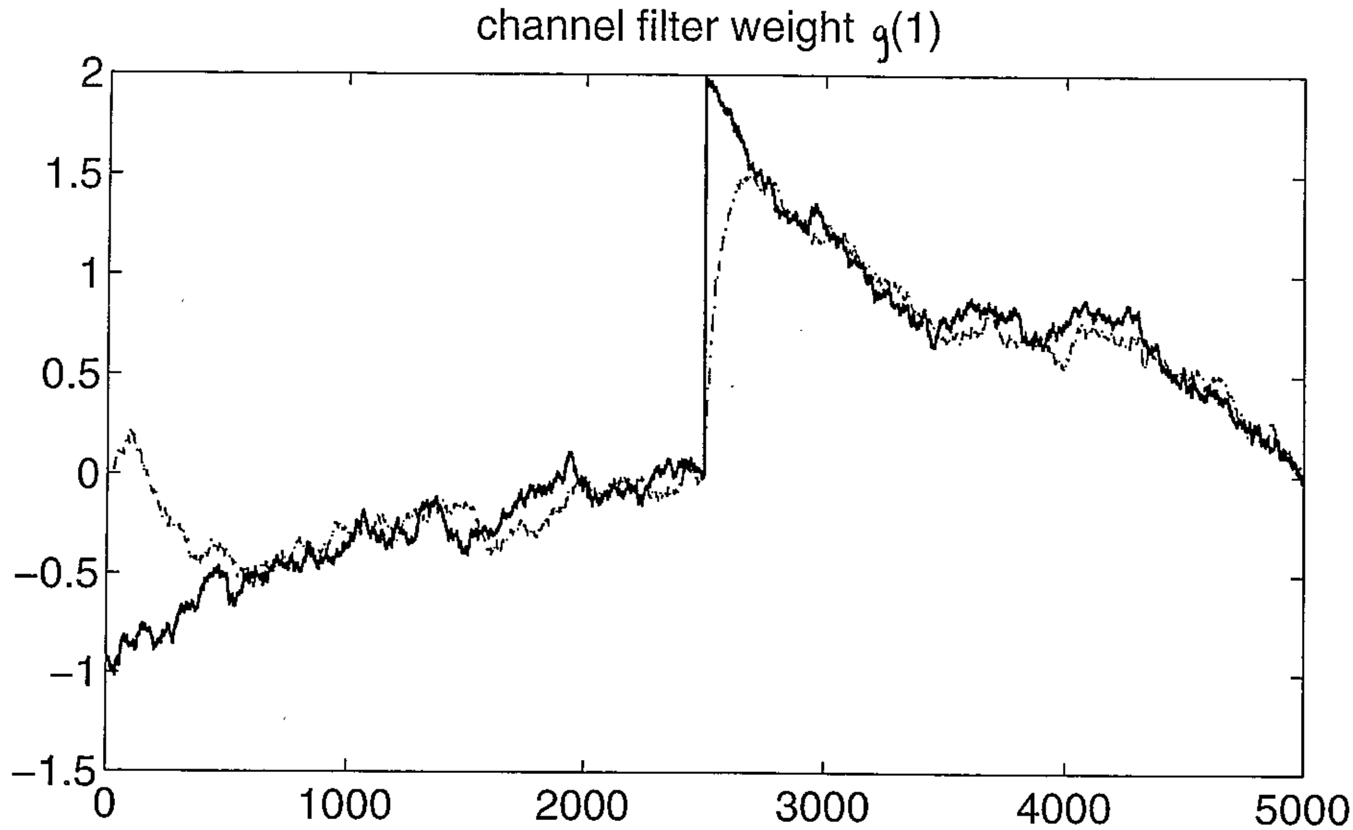
Input = square wave



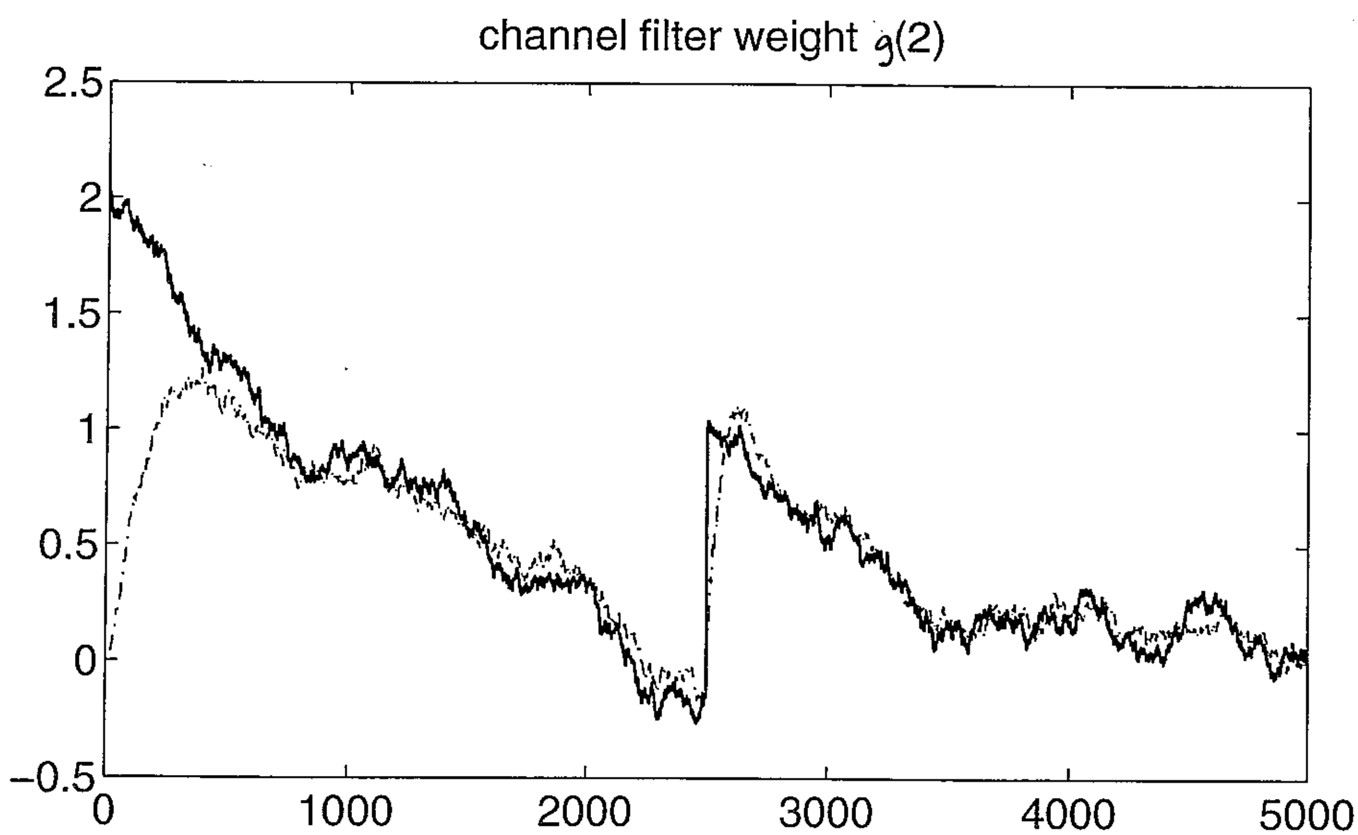




↑ filter changes drastically (e.g., car drives out of a tunnel)



channel filter weight g(1)
2
1.5
1
0.5
0
−0.5
−1
−1.5
0
1000
2000
3000
4000
5000



channel filter weight g(2)
2.5
2
1.5
1
0.5
0
−0.5
0
1000
2000
3000
4000
5000