BME 483, Introduction to MRI — 3/8/01
Douglas C. Noll — Handout #2

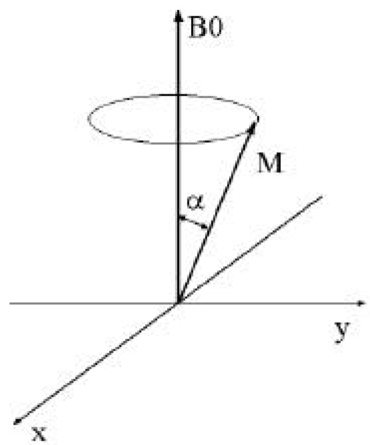
Homework #1
Due: 3/15/00

1. Determine the net nuclear magnetization of a 40 mM concentration of sodium (Na) ions at 1 T and at 310 K. 40 mM is the average concentration of Na in the human brain. Magnetization is the net magnetic dipole per unit volume. Your answer should have units A/m. Show your work.

   (Hint: 1 Molar concentration of some molecule (or ion) indicates that there is 1 mole of molecules (ions) per 1 liter of solvent (e.g., water). Units help: $J/T = Am^2$)

   Useful constants:
   k (Boltzmann's) = 1.38×10^{-23} J/K
   Avagadro's number = 6.02×10^{23} $(\text{mole})^{-1}$
   $\gamma = 2\pi \times 1.127 \times 10^7$ $(T\ s)^{-1}$ for ^{23}Na
   h (Plank's) = 6.63×10^{-34} J s ; h(bar) = $h/2\pi$

2. For a magnetization vector **M** and an applied field $\mathbf{B_0}$ and an angle α separating them, show that the rate of precession of **M** around $\mathbf{B_0}$ is $\omega_0 = \gamma B_0$. To show this, calculate the speed of the tip of **M** and divide by the path to precess around $\mathbf{B_0}$.



3. A material has equilibrium magnetization M_0 and relaxation time constants T_1 and T_2. If a $\pi/2$ excitation pulse is applied, find the expression for $|M(t)|$, the magnitude of the magnetization vector as a function of time. Show that if $T_2 < T_1$, $|M(t)|$ can never exceed M_0.

4. Consider two materials A and B with the same M_0, but with T_2 relaxation times (T_{1A}, T_{2A}) and (T_{1B}, T_{2B}). Let $\Delta s_{xy}(t) = M_{xyA}(t) - M_{xyB}(t)$ be the difference in transverse magnetization and $\Delta s_z(t) = M_{zA}(t) - M_{zB}(t)$ be the difference in longitudinal magnetization. Assume a $\pi/2$ excitation pulse. Find an expression for the time that maximizes each of the above differences.