BME 483, Introduction to MRI 3/20/01
Douglas C. Noll Handout #3

Homework #2

Due: 3/27/00

1. Consider a magnetic dipole, **M**, in the presence of an applied main magnetic field, **B**=B_0**k**, where **k** is the unit vector in the z-direction. Assume that initially, **M** is at equilibrium, has length M_0 and is aligned with the main magnetic field (along the z-direction). Describe (or sketch) the position of **M** in the rotating frame using a frame frequency of at $\omega_0 = \gamma B_0$ for the following sequence of events. Please ignore the effects of any T1 or T2 relaxation.
   a. At equilibrium.
   b. A rotating magnetic field of strength B_1 and rotational frequency ω_0 is applied for a period of time $\tau_1 = \pi/(2\,\gamma B_1)$.
   c. The main field is changed to **B**=$(B_0+\Delta B)$**k** for a period of time $\tau_2 = \pi/(\gamma\,\Delta B)$.
   d. A rotating magnetic field of strength B_1 and rotational frequency ω_0 is again applied for a period of time $\tau_3 = \pi/(2\,\gamma B_1)$. s

2. Determine the 1D Fourier Transform (FT) of the following:
   a. $\mathrm{rect}(x-b)$
   b. $\mathrm{rect}(ax)$
   c. $\mathrm{rect}(ax)\mathrm{rect}(ax)$
   d. $\mathrm{rect}(x)*\mathrm{rect}(x)*\mathrm{rect}(x)$ [* = 1D convolution]
   e. $\mathrm{sinc}(x)\cos(2\pi f_0 x)$
   f. $\exp(-\pi(x-x_0)^2)$

3. Determine the 2D FT of the following:
   a. $\mathrm{rect}(x)\mathrm{sinc}(y)$
   b. $\mathrm{sinc}(x-a)\mathrm{sinc}(y/b)$
   c. $\exp(-\pi(r/a)^2)$
   d. $\exp(-\pi r^2)**\exp(-\pi r^2)**\exp(-\pi r^2)$ [** = 2D convolution]
   e. $\exp(-\pi((x-x_0)^2+(y-y_0)^2)$

4. Consider the function $g(x) = \mathrm{sinc}^2(x/X)$. Determine its spectrum, $G(s)$. Next, determine the sampling frequency f_s that will prevent aliasing when sampling $g(x)$.

5. Consider the function $g(x,y) = \exp(-\pi(x^2+y^2)$ (a real and even function) which has the 2D-FT: $G(u,v) = \exp(-\pi(u^2+v^2)$. Describe (in words) what happens to the appearance of the function and of its spectrum (its 2DFT) for each of the following modifications:
   a. $g(x/a,y/a)$ for $a>1$
   b. $g(ax,ay)$ for $a>1$
   c. $g(x-a,y)$ for $a>0$
   d. $a\,g(x,y)$ for $a>0$
   e. $-g(x,y)$
   f. $g(x,y)\cos(2\pi f_0 x)$
   g. $g(x,y)\exp(i2\pi f_0 x)$