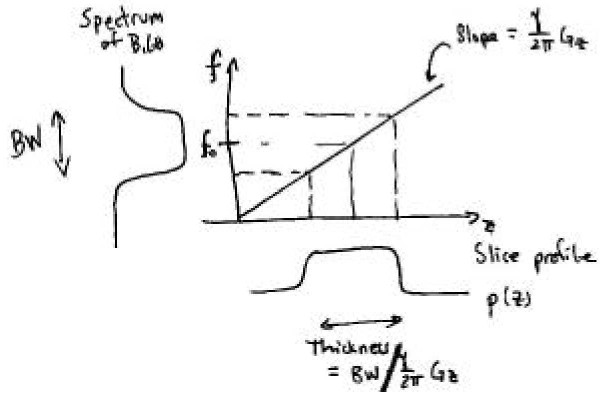
Imaging Notes, Part IV

Slice Selective Excitation

The most common approach for dealing with the 3rd (z) dimension is to use slice selective excitation. This is done by applying a z-gradient so that the resonance frequency varies in the z-direction and applying a bandpass RF pulse to excite only the those spins whose resonant frequency lies within the band:

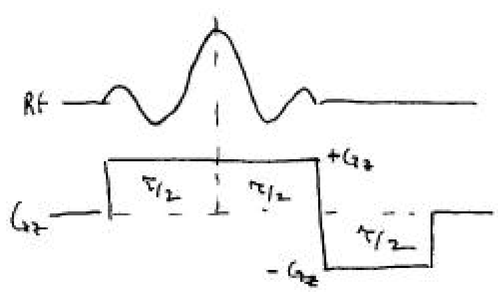


For a given RF pulse, $B_1(t)$, the slice profile (variation of $m_{xy,rot}$ with z) is related to the spectrum of the RF pulse. In the rotating frame, the slice profile and the magnetization will be:

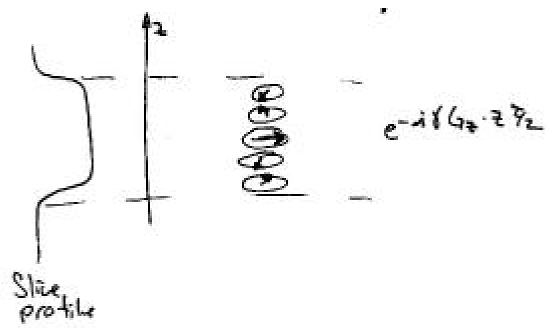
$$\left|m_{xy,rot}(z)\right| \propto \left|F\{B_1(t)\}\right|_{f=\frac{g}{2p}G_z z}$$

and $f = \frac{g}{2p} G_z z$ is the conversion between spectrum and the z location.

The pulse sequence to produce this slice selection is:

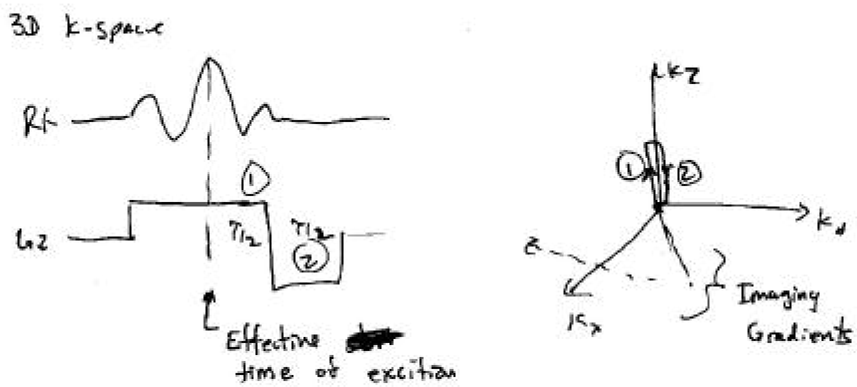


The negative gradient pulse after the RF pulse is over is necessary to correct for phase accumulation during the RF pulse. Conceptually, we think of the RF pulse occurring right at the center of the waveform (e.g. at t = t/2) and the positive z gradient during the second t/2 interval creates a phase distribution through the slice, $\exp(-igG_z zt\ /\ 2)$. Graphically, it looks like this:



which can lead to undesired phase destruction when integrated by the RF coil. How is this fixed? We simply apply a negative G_z for a period t/2. This is often called a slice rephasing pulse.

There is a k-space picture to this. For this, we again assume that the RF pulse occurs instantaneously a the center of the pulse (at t/2) and we begin accumulation area in k-z after that point. By applying a negative gradient for the same duration as the last half of the pulse, the areas cancel and the k-space location in the z direction is returned to the origin.



Observe the flip angle at the center of the slice is:

$$a = \int gB_1(t)dt = gF\{B_1(t)\}\ |_{x=0}$$

Example – the sinc RF pulse

Consider an RF pulse roughly in the form:

$$B_1(t) = A\text{sinc}\left(\frac{t}{T}\right)$$

which has a spectrum:

$$F\{B_1(t)\} = AT\text{rect}\left(\frac{f}{BW}\right), \text{ where } BW = 1/T$$

The slice profile will be:

$$p(z) = \text{rect}\left(\frac{z}{\Delta z}\right)$$

where the slice thickness is $\Delta z = \dfrac{2pBW}{gG_z}$ and flip angle is $a = gAT$.

Putting Slice Selection with the Signal Equation

Our slice profile function is:

$$p(z) \propto F\{B_1(t)\} \,\Big|_{f=\frac{g}{2p}G_z z}$$

Now we go back to the case where we have a 3D distribution of magnetization by substituting $im_0 = m(x,y,z)$ and putting it into the signal equation (again the RF coil integrates across the object):

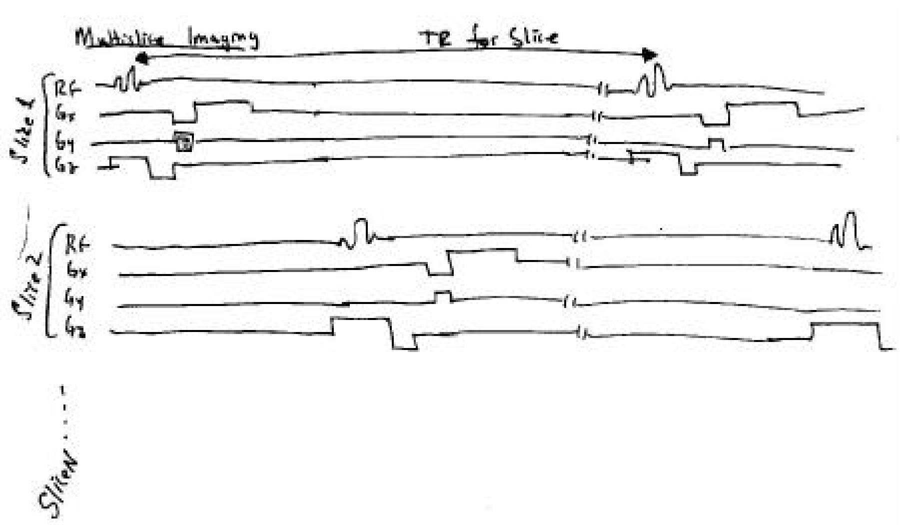
$$s(t) = \iiint m(x,y,z)\,p(z)\exp\left(-i2p\left(xk_x(t) + yk_y(t)\right)\right)dxdydz$$

Here we are performing 2D imaging while integrating across the slice profile.

Mutlislice Imaging

The most common way to image 3D volumes in MRI uses interleaved slice selective excitation. Here, slice 1 is excited and part of the k-space data are acquired, then slice 2 is excited and acquired, then slice 3 and so on. After all have been acquired, we come back to slice 1 to acquire additional parts of the k-space data, etc. When one slice is excited, the others are not perturbed and thus each slice has it's own T1 recovery time (TR). Slice selection allows the efficient use

acquiring many slices:



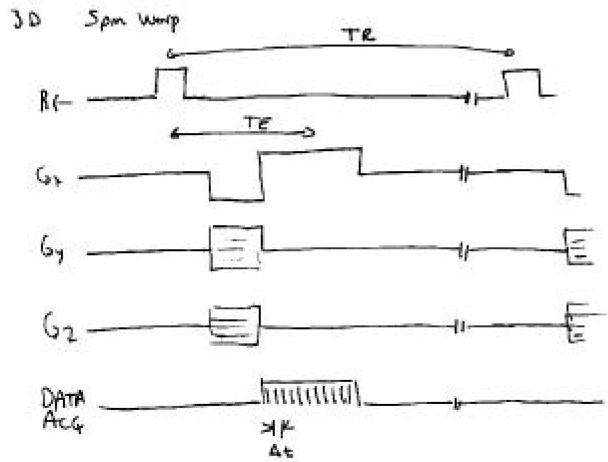
For a slice-selective, 2D spin-warp acquisition the overall acquisition time will be N_y*TR. For example, if we are interested in acquiring a T1-weighted image with 20 slices and a 500 ms TR and 128 phase encoding lines in k-space, the total acquisition time for these 20 slices is N_y*TR = ~ 1 minute.

The 3rd Dimension – Phase Encoding in Z

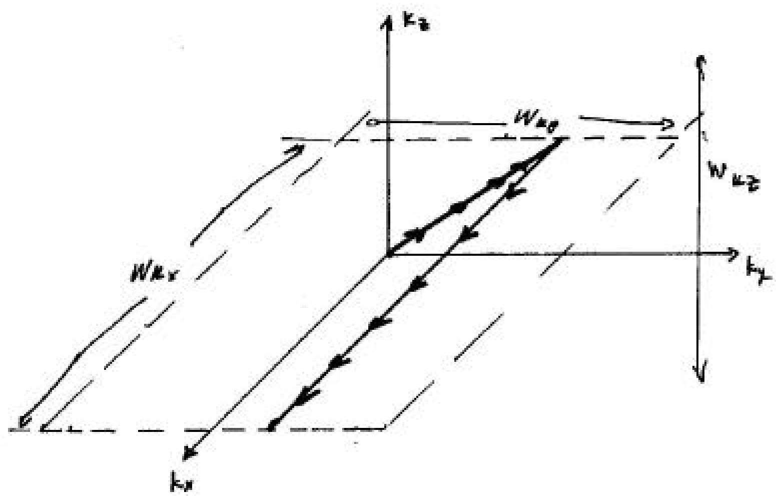
The 3D signal equation can be written as follows:

$$s(t) = \iiint m(x,y,z)\exp(-i2p(k_x(t)x + k_y(t)y + k_z(t)z)dxdydz$$
$$= M(u,v,w)\ \Big|_{u=k_x(t),v=k_y(t),w=k_z(t),}$$

where $M(u,v,w)$ is the 3D FT of $m(x,y,z)$. In the spin-warp method for 2D acquisition, one line at a time is acquired in the 2D Fourier domain (or k-space). This method is easily extended to 3D by using phase encoding in two dimensions (rather than 1) and frequency encoding in the remaining dimension:



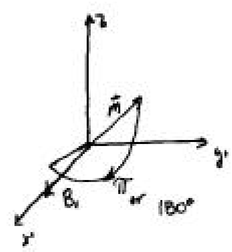
This results in the acquisition of a cubic data set one line at a time:



The sampling requirements and spatial resolution requirements are the same as they would be for the 2D spin-warp method ($FOV_z = 1/\Delta k_z$; $\Delta z = 1/W_{kz}$). If there are N_y and N_z samples in the y and z directions, respectively, then the total time to acquire the 3D volume is N_y*N_z*TR. For example, for $N_y = N_z$ =128 and TR = 33 ms, the overall image acquisition time is 9 min – rather long!

Spin Echo Pulses

We've described 180 degree RF pulses for purposes of inverting the m_z magnetization. Let's consider the effect of a 180 degree B1 pulse applied to the x' axis in the rotating frame on a magnetization vector, $\mathbf{m} = [m_x, m_y, m_z]$:



For t_{180}- and t_{180}+ being the time just before and after the 180 degree pulse, the magnetization will be:

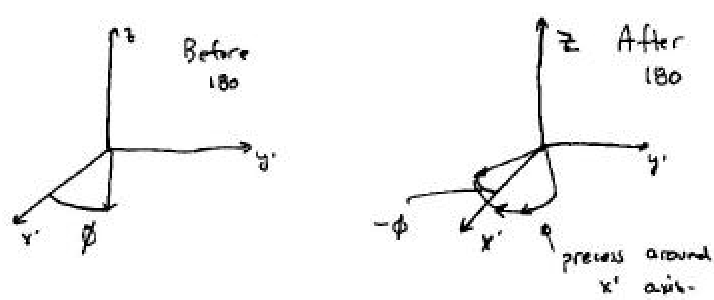
$$m_{x,rot}(t_{180}+) = m_{x,rot}(t_{180}-)$$
$$m_{y,rot}(t_{180}+) = -m_{y,rot}(t_{180}-)$$
$$m_{z,rot}(t_{180}+) = -m_{z,rot}(t_{180}-)$$

Now, let's look at a magnetization vector that is lying in the transverse plane (m_z = 0). Suppose the vector was originally positioned on the x' axis and phase, $f(x,y,z,t)$ has accumulated due to ΔB terms. The phase after the 180 degree pulse will be:

$$f(x,y,z,t_{180}+) = -f(x,y,z,t_{180}-)$$

or equivalently:

$$m_{xy,rot}(t_{180}+) = m_{xy,rot}(t_{180}-)^*$$



Why do spin-echo pulses?

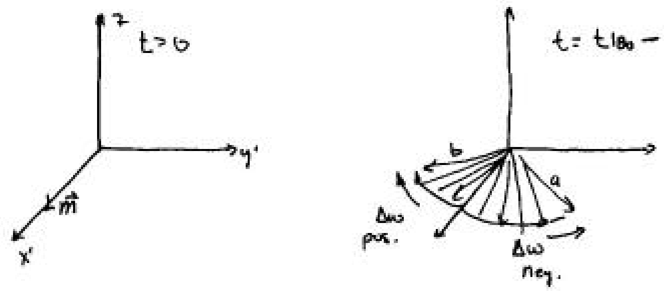
Magnetic field inhomogeneity can results in intra-voxel signal dephasing. Consider a magnetic field inhomogeneity function $\Delta B(x,y,z)$. The effective magnetic field (rotating frame) will be:

$$B_{z,\text{eff}} = \Delta B(x,y,z)$$

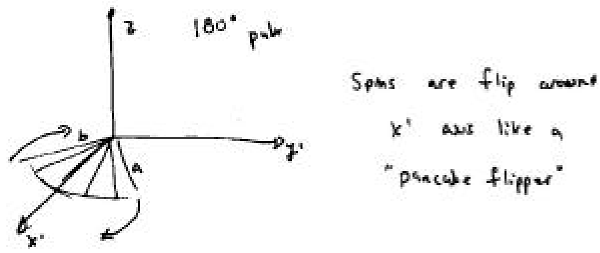
and the corresponding phase function is:

$$f(x,y,z,t) = \Delta w(x,y,z)t$$

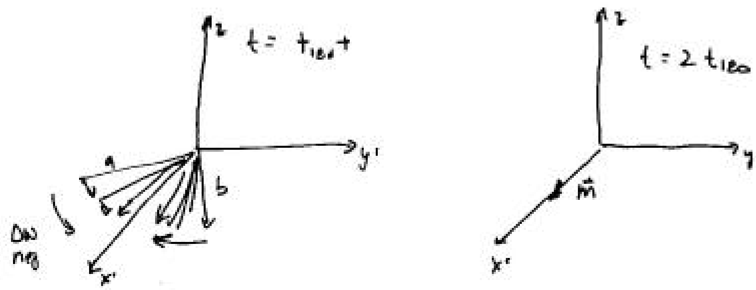
and when integrating across the voxel some signal may be lost. The spin-echo pulse brings this phase back together again. Consider the following example:



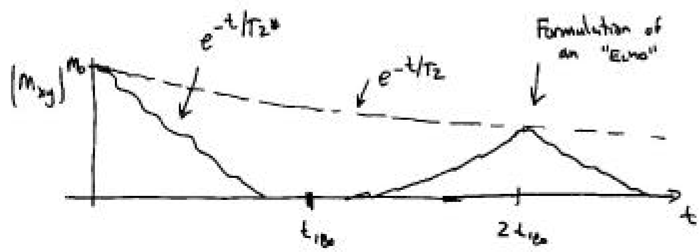
The phase accumulation at the time of the 180 is inverted by the 180 degree pulse:



At this point the phase continues to accumulate and if we look at time 2 t_{180}, we will have a total phase accumulation of exactly zero!



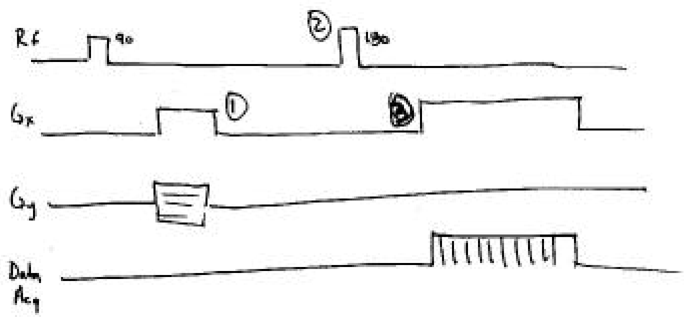
The size of the signal in the transverse plane ($|m_{xy}|$) will look like this:



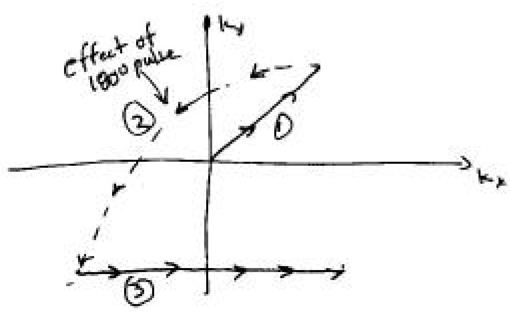
As shown here, the signal comes back together again in an "echo" at 2 t_{180}. The more rapid decay of the signal due to T2 decay plus inhomogeneity effects is given another decay term – T2*. When all dephasing is cancelled by the spin-echo, however, the T2 decay still remains.

Spin-echo Spin-warp Pulse Sequence

Consider this pulse sequence:



Here the 180 degree pulse in addition to reversion the phase of the spins, inverts the position in k-space as shown here:



Noise in MRI

Sources of noise in MRI

- Thermal noise from body – thermal vibration of ions, electrons, etc. [Dominant source of noise in most MRI systems]
- Quantization noise in the A/D devices
- Preamp/electronic noise
- Thermal noise in RF coil

Some comments on thermal noise:

- Not related to the NMR
    - Present with or without B_0, RF, Gradients
- Uniform spectral density (near w_0) – white
- Comes from the whole body – amount of noise depends on the amount of the body to which the receive coil is sensitive

The signal to noise ratio (SNR) can be determined from the following relationships

- The noise/pixel in a 2D image will be then be:

$$s_n^2 \propto \frac{1}{N_x N_y \Delta t} = \frac{1}{T_{A/D}}$$

  where N_x is the number of samples in the x-direction, N_y is the number of samples in the y-direction, Δt is the sampling time and $T_{A/D}$ is the total time the A/D is sampling.
- The signal is proportional to $m_0 V$ where $V = \Delta x \Delta y \Delta z$ is the "voxel" volume and Δz is the slice thickness.
- The signal to noise ratio is then:

$$SNR \propto \frac{signal}{s_n} = m_0 V \sqrt{T_{A/D}}$$

  (m_0 is proportional to r - the concentration the nucleus of interest, B_0, and g.)

Examples

Case 1: Suppose we find that we have an image that is too noisy, so we average together neighboring pixels to achieve $\Delta y' = \Delta y*2$ (all other dimensions remain the same and Δt hasn't changed either). Since by averaging in image domain, we effectively are discarding samples in k-space, $T_{A/D}' = T_{A/D}/2$ and:

$$SNR' = 2\Delta x \Delta y \sqrt{\frac{T_{A/D}}{2}} = \sqrt{2} SNR_{orig}$$

That is, we've improved the SNR by sqrt(2).

Case 2: Suppose we knew in advance that the SNR of an image was too noisy, and we compensated by acquiring a lower resolution $\Delta y' = \Delta y*2$ (all other dimensions remain the same) but we've compensated so as to preserve the original acquisition time $T_{A/D}' = T_{A/D}$. Thus:

$$SNR' = 2\Delta x \Delta y \sqrt{T_{A/D}} = 2SNR_{orig}$$

From these two examples, we see that it is preferable to anticipate the SNR that is necessary for a given image an set the acquisition accordingly. We don't achieve as good of an SNR by smoothing the image after it is acquired than if we had acquired at the appropriate resolution originally.