## Emission Computed Tomography Notes

Introduction

Emission computed tomography (ECT) is the CT applied to nuclear medicine. There are two varieties of ECT:

1. SPECT – single-photon emission computed tomography. This is the usual gamma ray sources an collection using a gamma camera.
2. PET – positron emission tomography. Here a positron is the nuclear decay product and positrons undergo mutual annihilation with an electron and *two* 511 keV photons are emitted.

SPECT

As in a gamma camera, a parallel hole collimator is used to acquire photons from a 3D source of photons. In comparison to the gamma camera, we will swap coordinate axes to match what we've done in CT:
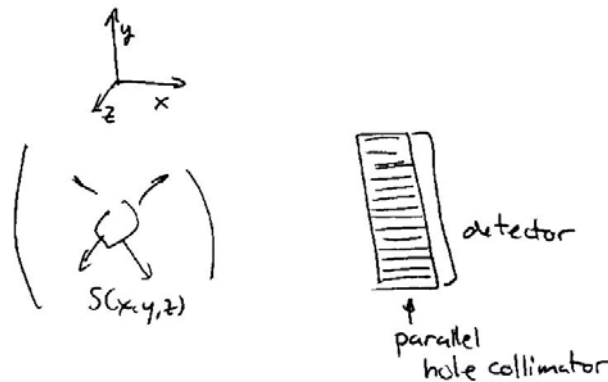
Here, the image intensity for a projection in x will be:

$$I_d(y, z) = \int s(x, y, z)dx$$

(we've dropped the capture efficiency constant). As we can see, we simultaneously acquire projections for many z planes. Now, we move the detector around the object to gather projections at many different angle:
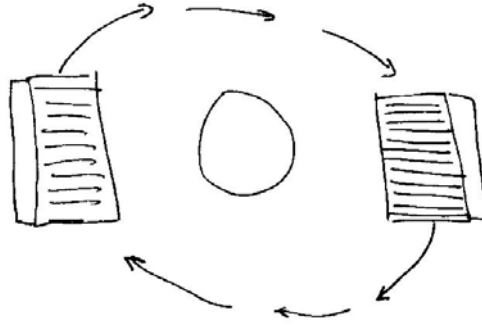
For efficiency, we might have 2-4 detectors acquiring simultaneously. We replace the *y* coordinate with *R*, the dimension along the projection:

$$g_\theta(R, z) = \int s(x, y, z)\delta(x\cos\theta + y\sin\theta - R)dxdy$$

For each z-plane, we now can reconstruct the images using the techniques learned for CT.

Attenuation issues. Because there is attenuation as the photons pass through the body, there might be artifact in reconstructing the image of the source. To look at this issue, we again examine the projection in x-direction:

$$g_x(R, z) = \int s(x, y, z)\exp\left(-\int_{-r_0}^{x} \mu(x', y, z)dx'\right)dx$$

where r_0 is the radius of the body being imaged. since we are acquiring at many different angles, we can also acquire in the opposite direction (e.g. acquire projections at $\theta = 0$ and $\theta = \pi$). The complementary projection is:

$$g_{-x}(R, z) = \int s(x, y, z)\exp\left(-\int_{x}^{r_0} \mu(x', y, z)dx'\right)dx$$

For a point source at $x = a$, $s(x,y,z) = \delta(x-a)$, and a uniform μ, we get:

$$g_x(R, z) = \int \delta(x - a)\exp\left(-\int_{-r_0}^{x} \mu(x', y, z)dx'\right)dx$$
$$= \int \delta(x - a)\exp(-\mu(a + r_0))dx$$
$$= \exp(-\mu(a + r_0))$$

and for the complementary projection it is:

$$g_{-x}(R,z) = \int \delta(x-a)\exp\left(-\int_{x}^{r_0}\mu(x',y,z)dx'\right)dx$$
$$= \int \delta(x-a)\exp(-\mu(r_0-a))dx$$
$$= \exp(-\mu(r_0-a))$$

By averaging these two projections we get:

$$\bar{g}_x(R,z) = \tfrac{1}{2}\left[\exp(-\mu(a+r_0))+\exp(-\mu(r_0-a))\right]$$
$$\approx \tfrac{1}{2}\left[(1-\mu(a+r_0))+(1-\mu(r_0-a))\right]$$
$$= 1-\mu r_0$$

which is independent of the position of the source along the projection (*a*), though not independent of attenuation. So, to first order, the depth dependent attenuation effects are canceled by average projections in opposite directions. This is standard practice in SPECT.

Comments:

1. SPECT collects parallel ray geometry projections (in projection x-ray imaging, it is a fan beam geometry).

2. Multiple parallel slices are acquired simultaneously.

3. Projections are acquired over the full 2π range (rather than π) in order to partially cancel attenuation effects.

4. Multiple camera heads are used to improve efficiency (number of photons captured is proportional to number of heads).

5. Capture efficiency is still quite low and SNR/spatial resolution is usually poor.

PET

In positron emission tomography, an accelerator is used to create unstable nuclei that emit positrons. The most common positron emitters used are:

Decay Event	Half Life
$^{11}C \rightarrow {}^{11}B + \beta+$	20 min.
$^{15}O \rightarrow {}^{15}N + \beta+$	2 min.
$^{18}F \rightarrow {}^{18}O + \beta+$	2 hrs.

The positrons undergo mutual annihilation with an electron and *two* 511 keV photons are emitted. What makes PET unique from SPECT is that the two photons are emitted in nearly equal and opposite directions. This results from conservation of momentum and energy. To the extent that they are not exactly opposite in direction results from some initial momentum on the positron:
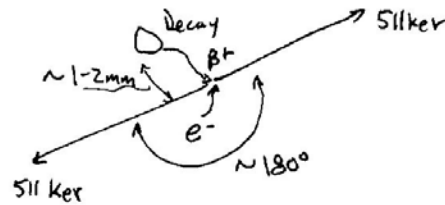

The rest mass of both the positron and the electron being annihilated are both 511 keV and this is all converted to photon energy.

The photons are captured by a ring of detectors and an electronic circuit (*coincidence detector*) analyzes the results. If two photons are captured closely spaced in time, the coincidence detector will identify these as coming from the same decay event. Based on the geometry of the detectors, we can say with a high degree of certainty that the decay event occurred along a particular line and a counter for that line is incremented to indicate that an event has been detected.
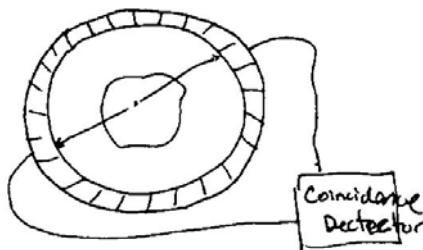
Pairs of detectors form a line. Each detector may detect events along many lines. A group of parallel lines can be used to form a projection at a particular angle and a different group of parallel lines can be used to form a projection at a different angle:

Detector Pairings for Different Projections
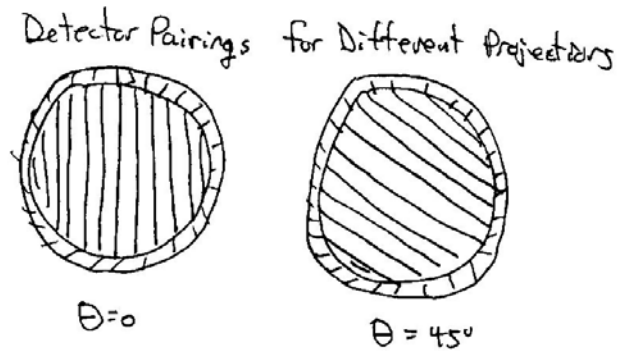
These projects can then be reconstructed by the usual methods.

Comments:

1. In PET projection geometry is determined by pairs of detectors that detect events. In SPECT, this is determined by the parallel hole collimator. In x-ray CT, it is determined by the line from the source to the detector.

2. There is no collimator so the capture efficiency is much higher than SPECT (typically 2% (in 2D mode) vs. 0.02% in SPECT).

3. The higher E (511 keV vs. the typical 140 keV in SPECT) results in less attenuation.

4. Higher resolution is possible compared to SPECT.

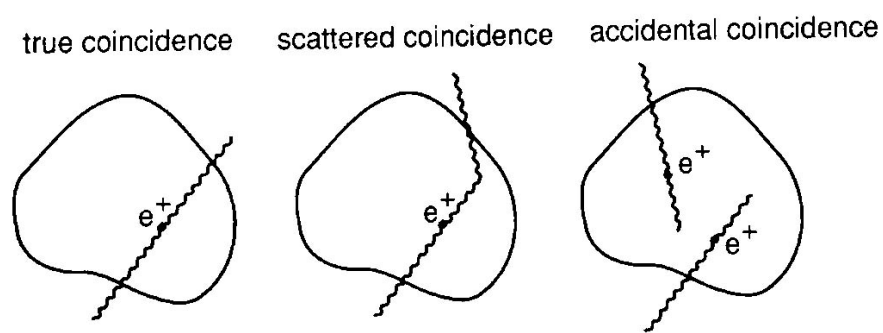5. Random (accidental) and scattered coincidences are sources of error:

Figure 1: The three types of coincident events detected by a PET scanner

6. The 3D dimension is acquired by having a stack of detector rings. In 2D mode, septa prevent decays from different z positions from striking the wrong ring.
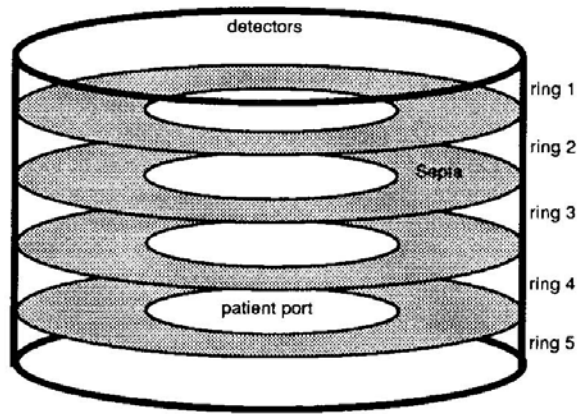

Figure 3: Schematic representation of a 5-ring cylindrical scanner (thick line) with annular interplane septa (grey) and the central patient entry port. The distance between the rings is increased for clarity
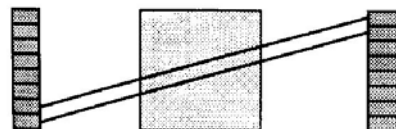
7. There is a true 3D acquisition in which the septa are removed. This leads to a dramatic increase in capture efficiency (and thus SNR). Here we have projections in the x-y plane as well as projections tilted in the z-direction. The image reconstruction problem for a set of 3D projections is very interesting, but beyond the scope of this course.
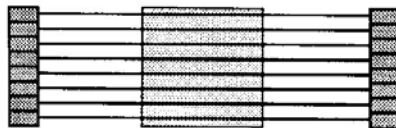
Ring difference = 0
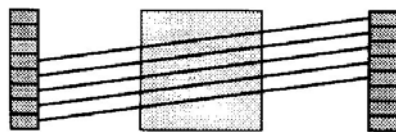
Ring difference = 3

Ring difference = 6