X-Ray Notes, Part I

X-ray Imaging

Images are characterized by the interaction of x-ray photons and tissue.

Physics

Definition: Radiation – a stream of particles or photons.

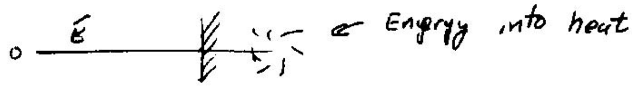
Particles: α ($^{2+}$He), e^- (electrons), β (electrons emitted from nuclei),

β^+ (positrons), p^+ (proton), n^0 (neutrons)

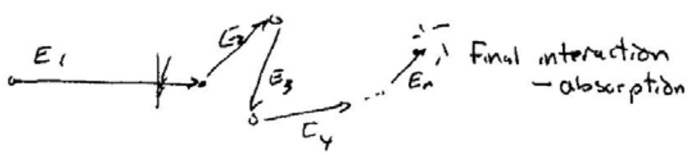
Photons: x-ray, γ, annihilation photons, etc.

Models for interaction of radiation and matter:

1. Absorption (generally low kinetic energy (KE))



2. Scattering



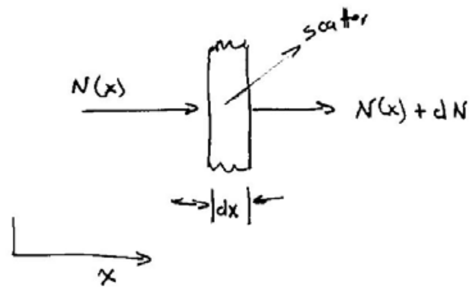
3. Not a typical interaction – a gradual loss of energy

The charged particles above (α, e^-, β, β^+, p^+) interact very strongly with tissue and typically do not pass completely through the human body and thus cannot be used for imaging. Of the above particles photons and neutrons(n^0) pass through the body with an appropriate amount of interaction for imaging (too little is also bad).

Behavior of Radiation Along a Line

Assumptions:

1. Matter consists of discrete particles separated by distances that are large compared to the size of the particles.
2. For a given path length along a line, an x-ray photon either interacts (with prob. *p*) or it doesn't and all interactions are independent.
3. Scattered photons scatter at a different angle and don't contribute to the continuing flux of photons along the line.



The change in the number of photons is:

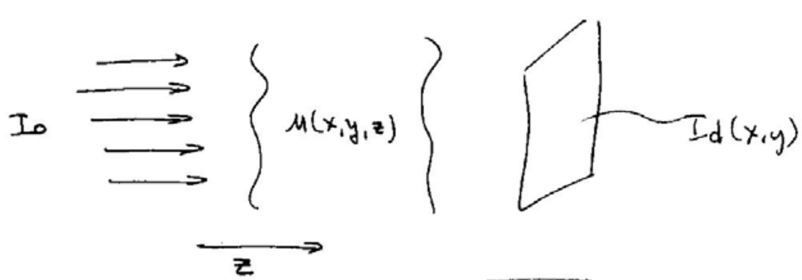
$$dN \propto -N(x)dx$$
$$dN = -\mu N(x)dx$$
$$\frac{dN}{dx} = -\mu N(x)$$
$$N(x) = N(0)\exp\left(-\int_0^x \mu(x')dx'\right)$$

were μ is the "linear attenuation coefficient" and has units (distance)$^{-1}$. For a constant μ:

$$N(x) = N(0)\exp(-\mu x)$$

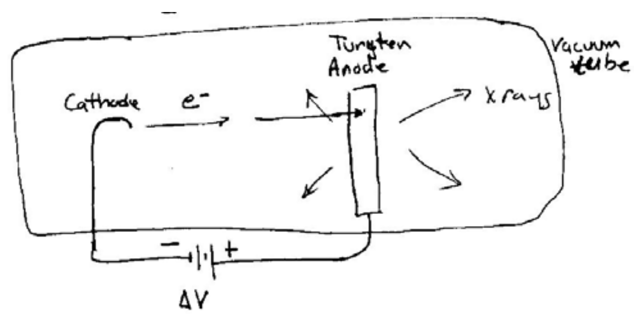
The Basic X-ray Imaging System

Now consider a parallel ray x-ray flux that has intensity I_0 (intensity is photons/unit area/unit time) the passes through a 3D object having a distribution of attenuation coefficients $\mu(x,y,z)$ and projects to an image $I_d(x,y)$:



$$I_d(x,y) = I_0 \exp\left(-\int \mu(x,y,z)dz\right)$$

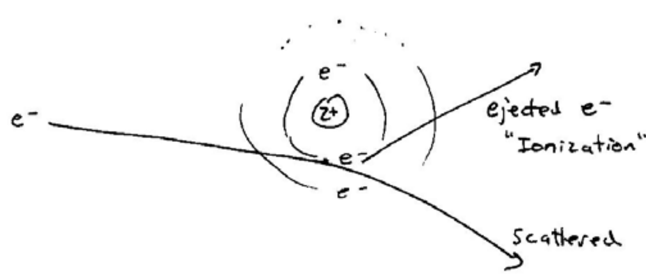
Generation of x-rays



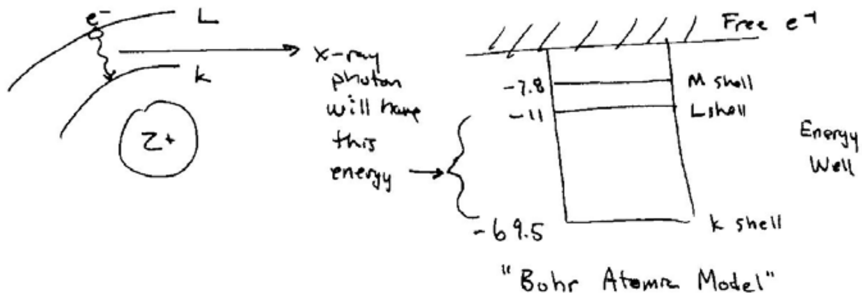
- Target is usually a high-Z, heavy element – typically W, tungsten.
- Electrons are accelerated by the voltage between the cathode and the anode.
- A potential energy of E=qΔv (e.g. e * 150 kV = 150 keV) all gets converted to kinitic energy E = ½ $m_e v^2$ (e.g. also 150 keV).

Kinds of electron interactions:

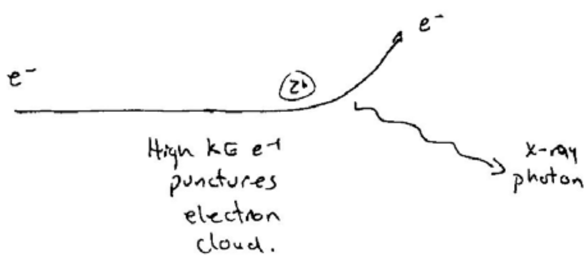
a. Inelastic (energy absorbing) scattering with atomic electrons – the ejection of a bound electron followed by emission of a photons from spontaneous energy state transitions.



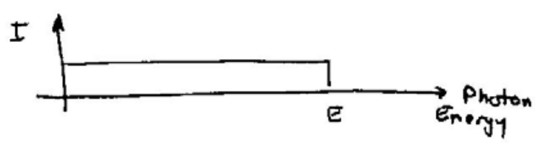
The Bohr model accounts for absorption/generation of discrete valued energies.
58.5 keV is one "characteristic" x-ray for W. Any combination of shell transition energies will also be characteristic energies (e.g. 3.2 and 61.7 keV). Very low energies are hard to observe due to other absorption processes.



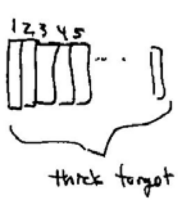
b. Bremsstrahlung "Braking" Radiation – Acceleration (change in direction) of electron by Coulomb attraction to the large, positively charged nucleus leads to the generation of photons (acceleration of any charged particle will do this).

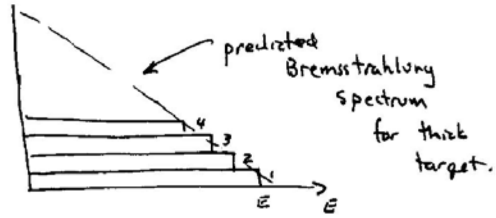


For electrons of a particular energy, E, striking an infinitely thin target, Bremsstrahlung radiation will have a uniform distribution of energy between 0 and E.



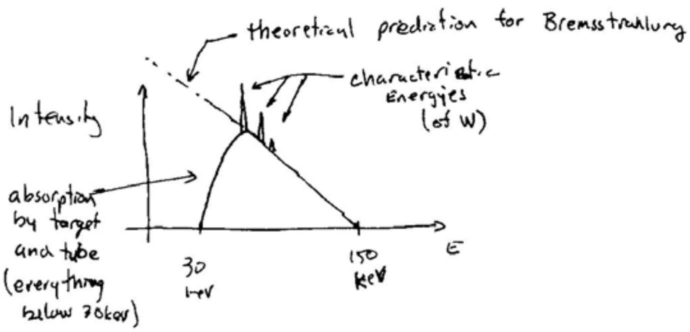
We assume that all electrons interact. For a thick target, it is often modeled as a series of thin targets where the highest energy impinging upon subsequent stages is reduced by the interactions. Each thin target produces a new uniform spectrum, but with a lower peak energy. The resultant spectrum is approximately linear from a peak at 0 keV to 0 at E.





The x-ray Spectrum

- For electrons with energy E, the maximum x-ray photon energy is E.
- $E = h\upsilon = \dfrac{hc}{\lambda}$
- Very low energy photons are absorbed by the target and by the glass in the x-ray tube.
- Spectrum will have a combination of Bohr (discrete) energies and Bremsstrahlung radiation:

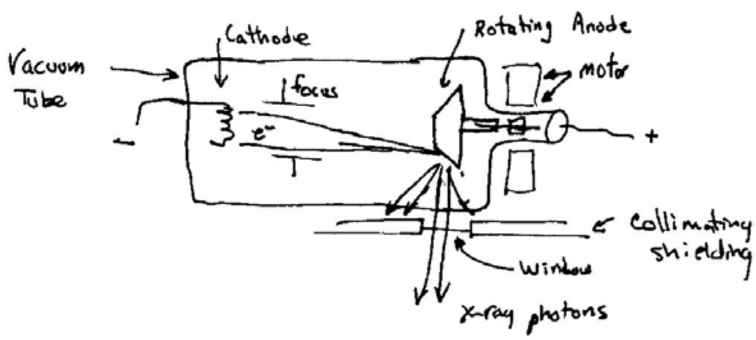


- The x-ray spectrum is function of photon energy: $I_0 = I_0(E)$
- I now represents energy/unit time/unit area or power/unit area.

Practical x-ray tube

Why Tungsten?

- x-ray spectrum in desired range
- High Z (high efficiency in stopping electrons)
- High melting point (3300 deg. C) – typical operation temp is ~2500 deg. C – this is due to the low efficiency of the electron to x-ray conversion (~0.8%). The rest goes into heat.
- Example:



- Rotation of target to reduce peak temp

- Shielding to collimate beam
- Window further filters x-ray spectrum ("hardens beam" – makes it have a higher average E)

The Attenuation Coefficient

We say above that the x-ray spectrum is a function of photon energy E: $I_0 = I_0$(E). The attenuation function is also a function of E: $\mu = \mu(x,y,z,\mathrm{E})$. The new expression for the intensity at the output will not be:

$$I_d(x,y) = \int_E I_0(E)\exp\left(-\int \mu(x,y,z,E)dz\right)dE$$

Note: I_d tells us nothing about z or E – it only gives us x,y information.

The x-ray attenuation coefficient μ is, of course, also a function of material properties. Two of the most important properties that affect the attenuation coefficient are tissue density, ρ, and the atomic number Z. As most x-ray photon/tissue interactions are photon/electron interactions both ρ and Z will influence μ.

For x-ray photons, there are 4 main types of interactions (listed in order of increasing likelihood with increasing photon energy, E):

1. Rayleigh-Thompson Scattering
2. Photoelectric Absorption
3. Compton Scattering
4. Pair Production

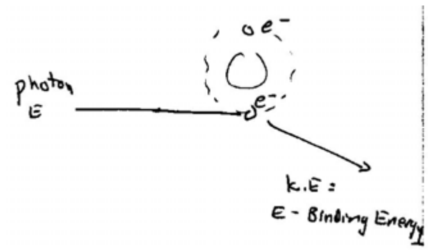
In general, we can write an expression for the attenuation coefficient as the some of these constiuent parts:

$$\mu(E) = \mu_{rt}(E) + \mu_{pe}(E) + \mu_{cs}(E) + \mu_{pp}(E) + \ldots$$

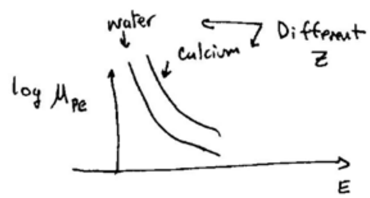
1. Rayleigh-Thompson Scattering or "coherent" scattering – atomic absorption with spontaneous emission at the same energy E. This is the same effect as is seen in x-ray

diffraction in crystals. This term is rarely important in the diagnostic energy range (50-200 keV).

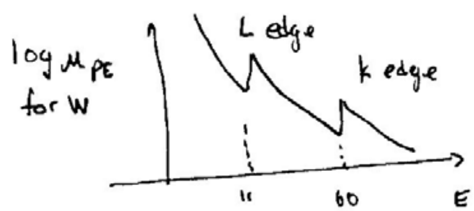
2. Photoelectric Absorption – Absorption of photon to ionize and eject an atomic electron. The ejected electron will have an kinetic energy of the photon energy less the binding energy of the electron.



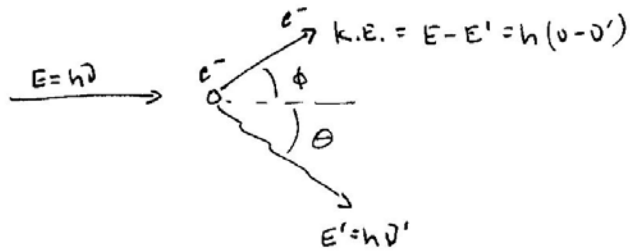
The photoelectric effect increases rapidly with atomic number, Z, and with decreasing energy. The photoelectric effect dominates μ in the lower part of the diagnostic spectrum.



For high Z materials (e.g. Lead, Iodine, Tungsten), the shell energy boundaries are evident in the μ vs. E plots. When the energy gets high enough to make that shell's electrons available to the PE effect (when E exceeds the binding energy), then the probability of a PE interaction increases.



3. Compton Scattering – scatting of photons by an elastic collision with a free electron. Elastic collisions preserve E and momentum (p). For loosely bound electrons or very high energy photons, the equations for free electrons hold reasonably well.



Unknowns: ϕ, θ, E', K.E.

Conservation of energy:

$$\text{K.E.} = E - E' = (m - m_0)c^2$$

where $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$ is the relativistic mass of the electron

Just a check on this equation … for $v^2 << c^2$, then

$$(m - m_0)c^2 = m(1 - \sqrt{1 - v^2/c^2})c^2$$

$$\approx m(1 - (1 - \frac{1}{2}\frac{v^2}{c^2}))c^2 = \frac{1}{2}mv^2$$

Conservation of momentum in x and y directions:

$$\frac{E}{c} = \frac{E'}{c}\cos\theta + mv\cos\phi$$

$$\frac{E'}{c}\sin\theta = mv\sin\phi$$

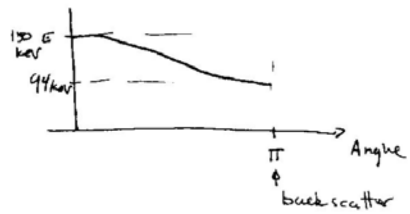
solving these equations we get the energy of the scattered photon:

$$E' = \frac{E}{1 + \frac{E}{E_e}(1 - \cos\theta)}$$

where $E_e = m_0c^2 = 511$ keV, the rest energy of an electron.

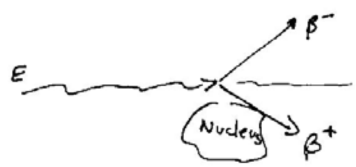
Comments:

- For $E << E_e$, there is very little change in energy with angle.
- For higher E:



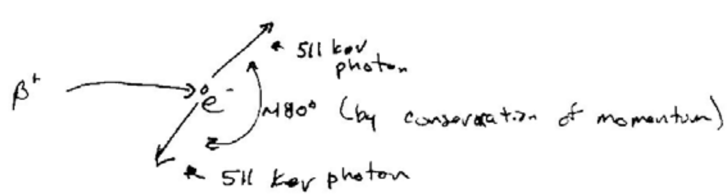
- For low *E*, scatter is essentially isotropic in angle
- For higher *E*, scatter is preferentially forward scattered (where there is very little change in photon *E*).
- It is very hard to discriminate between forward scattered photons and unimpeded photons based on energy.
- μ_{cs} is nearly constant across diagnostic spectrum
- Compton scatter comes mostly from atomic electrons (μ_{cs} is proportional to ρ)
- At higher *E*, Compton scatter dominates over the PE effect (most important effect in x-ray imaging).

4. Pair Production – the spontaneous creation of an electron/positron pair:



$$E' = E - 2\,(m_0c^2)$$

In this interaction, photon energy in transferred to mass energy in the electron and positron. Since the rest energy of each is 511 keV, pair production cannot occur for x-ray photons below 1022 keV (not in the diagnostic spectrum). Positrons will wander around until they bump into an electron, which will result in mutual annihilation and the emission of two 511 keV photons:



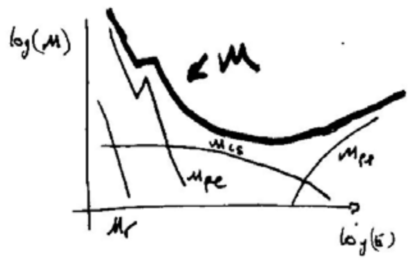
The ejected photons from a positron/electron annihilation is the basis for positron emission tomography [more on this later].

Total Linear attenuation coefficient for photons

Again, the combined coefficient is:

$$\mu(E) = \mu_{rt}(E) + \mu_{pe}(E) + \mu_{cs}(E) + \mu_{pp}(E) + \ldots$$

For example, the combined coefficient for lead is:



An alternate to linear attenuation coefficient is the “mass attenuation coefficient” which is defined as:

$$\tau = \mu / \rho \text{ (units: cm}^2\text{/gm)}$$

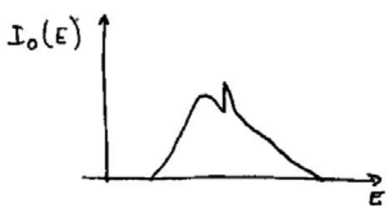
This parameter is convenient when describing the behavior of composite materials with N constituent components:

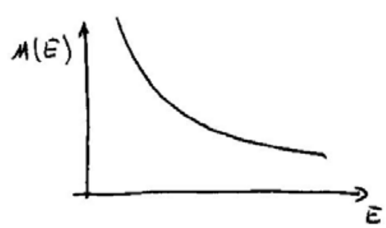
$$\tau = \frac{1}{M}\sum_{i=1}^{N} m_i \tau_i$$

where m_i are the masses of the components and M is the total mass.

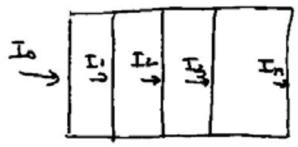
Beam Hardening

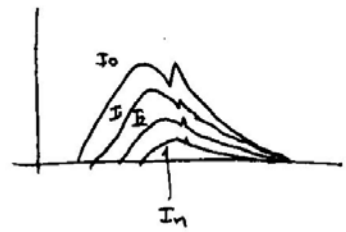
Because the attenuation spectrum is not uniform across the diagnostic energy spectrum, the output spectrum will have a different intensity distribution than the input spectrum, $I_0(E)$.





If we split an object into several smaller parts, and look at then energy spectrum at for each part:





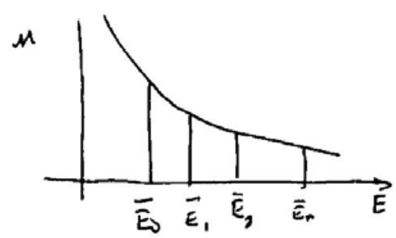
we will find that the mean energy:

$$\overline{E} = \frac{\int EI(E)dE}{\int I(E)dE}$$

will increase (get harder) as we move through the object:

$$\overline{E}_0 < \overline{E}_1 < \overline{E}_2 < ... < \overline{E}_n \,.$$

For medical imaging, this has the unfortunate consequence that a particular tissue type will have a μ that changes as a function of position along the path.



In particular, as we move deeper into the object, we will find that there is less attentuation than expected, given the initial spectrum, $I_0(E)$.

One solution is to make the beam "hard" to begin with. This is often accomplished by filtering out the low E photons with a thin metal plate (often use aluminum).