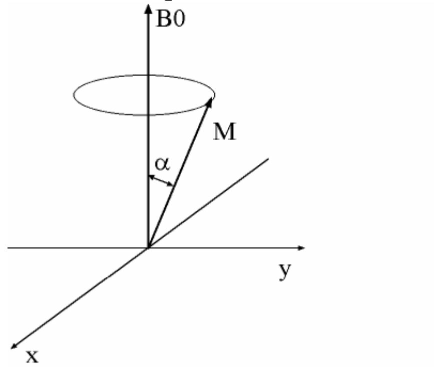
Homework #4

Due Date: Nov. 4, 2004

1. For magnetization vector **M** and applied field $\mathbf{B_0}$ with an angle α separating them, use the Bloch equations to show that the rate of precession of **M** around $\mathbf{B_0}$ is $\omega_0 = \gamma B_0$.



2. In class, we described quantum mechanical and classical properties of nuclear spin for the hydrogen nucleus, ^{1}H. Let's look at what changes if we are interested in imaging carbon-13, ^{13}C. Assume that $\gamma_C = \frac{1}{4}\gamma_H$. Please describe the following for carbon in terms of the equivalent terms for hydrogen. Assume all other factors (B_0, temperature, etc.) are the same.
   a. The difference in energy between spin-up and spin-down states, ΔE. (Please describe ΔE for ^{13}C in terms of the ΔE for ^{1}H.)
   b. The resonant frequency in the presence of applied field B_0, ω_0.
   c. The number of excess nuclei that are spin-up vs. spin-down, N_{diff}.
   d. The length of the magnetic dipole for a single nuclear spin, $|\mu|$.
   e. The size of the equilibrium net magnetic dipole for 1 mole of the substance, $|\mathbf{m}|$.

3. Consider a magnetic dipole, **M**, in the presence of an applied main magnetic field, $\mathbf{B}=B_0\mathbf{k}$, where **k** is the unit vector in the z-direction. Assume that initially, **M** is at equilibrium, has length M_0 and is aligned with the main magnetic field (along the z-direction). Describe (or sketch) the position of **M** in a frame rotating at $\omega_0 = \gamma B_0$ for the following sequence of events. Please ignore the effects of any T1 or T2 relaxation.
   a. At equilibrium.
   b. A rotating magnetic field of strength B_1 and rotational frequency ω_0 is applied for a period of time $\tau = 2\pi/(4\gamma B_1)$.
   c. The main field is changed to $\mathbf{B}=(B_0+\Delta B)\mathbf{k}$ for a period of time $T = \pi/(\gamma \Delta B)$.
   d. A rotating magnetic field of strength B_1 and rotational frequency ω_0 is again applied for a period of time $\tau = 2\pi/(4\gamma B_1)$. (Applied in the same direction as in b.)

4. Consider two materials, A and B with relaxation constants (T_{1A}, T_{2A}) and (T_{1B}, T_{2B}) respectively. Assume a 90 degree flip angle and at A, B have the same proton density.
   a. Determine the time that maximizes $\left\| M_{xy,A}(t) \right| - \left| M_{xy,B}(t) \right\|$.
   b. Determine the time that maximizes $\left| M_{z,A}(t) - M_{z,B}(t) \right|$.
   c. Assuming these materials was white matter and gray matter at 1.5 T, determine the repetition time (TR) that maximizes T1 contrast with a TE = 0 and determine the echo time (TE) that maximized T2 contrast for TR = ∞.