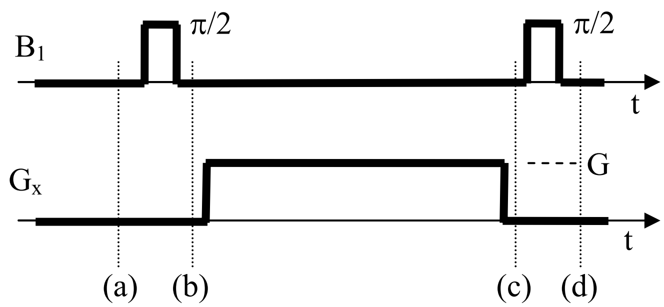
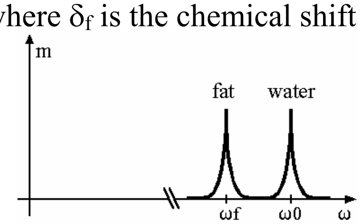
Homework #5
Due: 11/16/04

1. Consider an object with initial magnetization $m_0(x,y) = \mathrm{rect}(x/X, y/Y)$.
   a. Determine the 2D Fourier transform of $m_0(x,y)$.
   b. For gradient waveforms $G_x(t) = A$ and $G_y(t) = 0$, determine and sketch the k-space path and give an expression for the received signal, $s(t)$.
   c. For gradient waveforms $G_x(t) = a/X$ and $G_y(t) = a/Y$, determine and sketch the k-space path and give an expression for the received signal, $s(t)$.
   d. For gradient waveforms $G_x(t) = \frac{a}{X} rect\left(\frac{t-3T/2}{T}\right)$ and $G_y(t) = \frac{a}{Y} rect\left(\frac{t-T/2}{T}\right)$, where $T = \frac{4\pi}{a\gamma}$, determine and sketch the k-space path and give an expression for the received signal, $s(t)$.

2. Special pulse sequences can be used to generate usual patterns in the transverse or longitudinal magnetization across the image. We examine one such pulse sequence here. Using a 1D object $m_0(x) = 1$ that is initially in the equilibrium state, describe the longitudinal, $m_z(x)$, and transverse magnetization, $m_{xy,rot}(x)$, as functions of x at the points labeled (a)-(d). You may neglect relaxation and assume the duration of the gradient is T.



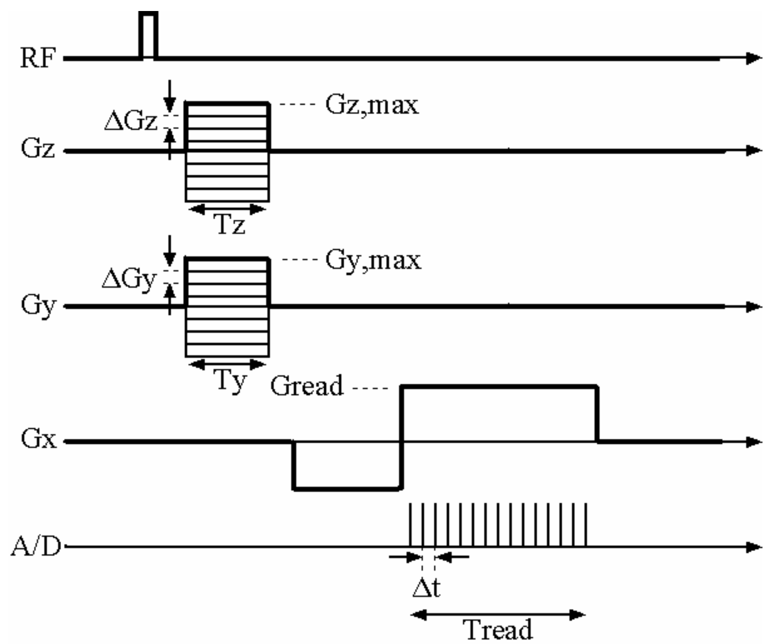
3. In class, we excited a particular slice by applying a gradient in the z-direction applying a band-limited RF pulse with a particular frequency of excitation and a particular bandwidth. Let look at a similar situation – excitation of particular chemical species, for example, water and fat. Different chemical species have different resonant frequencies based on a phenomenon known as chemical shift, in which surrounding electron clouds influence the strength of the B field seen by the nucleus. If we consider water is at some frequency $\omega_0 = \gamma B_0$, fat is at $\omega_f = \gamma B_0(1-\delta_f)$, where δ_f is the chemical shift of fat relative to water (3.5×10^{-6}).



   a. For $B_0 = 1.5$ T, find $(f_0 - f_f)$ Assume we are imaging ^1H.

b. Describe an RF pulse (pulse envelope shape and parameters, amplitude, carrier frequency, etc.) that has the appropriate characteristics for a 90 degree excitation of water but not fat. One common approach is the make the spectrum of the RF pulse symmetrical around the water resonance.

4. Consider a 3D spin-warp pulse sequence as shown in the sketch. Let $T_y = T_z = 5$ ms, and $T_{read} = 20$ ms. Suppose our desired field of views are $FOV_x = FOV_y = FOV_z = 20$ cm and $\Delta x = 1$ mm, $\Delta y = 2$ mm, and $\Delta z = 5$ mm. Assume γ for protons. Determine the following parameters:
   a. ΔG_z
   b. $G_{z,max}$
   c. ΔG_y
   d. $G_{y,max}$
   e. G_{read}
   f. Δt



5. Consider a volume coil and a surface coil. Let the volume coil have sensitivity, $S_v(x) = 1$, and the surface coil have the following sensitivity pattern (as a function of distance from the coil):

$$S_s(x) = \frac{1}{\left(1 + \left(\frac{x}{a}\right)^2\right)^{3/2}}, \text{ where } a \text{ is the coil radius.}$$

Let the noise variance of the volume coil be $\sigma_v^2 = 1$ and the noise variance of the surface coil be $\sigma_s^2 = 0.001 \cdot a^3$, where a is assumed to be in units of cm.
   a. For $a = 5$ cm, determine for which distance from the object surface it is advantageous (from a signal to noise ratio standpoint) to use the surface coil over the volume coil (and vice versa). SNR = (signal magnitude)/σ, where σ is the noise standard deviation.
   b. For $a = 10$ cm, determine for which distance from the object surface it is advantageous to use the surface coil over the volume coil (and vice versa).