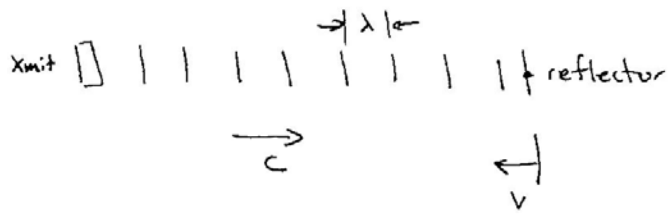
Ultrasound Notes 4: Doppler

Imaging of Blood Flow with US

One of the more interesting and useful applications of US is for the imaging of flow in the body. One way to do this is track speckle patterns over time using a cross-correlation technique. Another way to do this is take advantage of the Doppler effect. This is discussed below.

The Doppler Effect

In US, the Doppler effect has two components. We look first at the effective frequency seen by a reflector traveling towards the transmitter at velocity v.



During an interval of duration T, the number of cycles received will be $f_0 T$ plus the number of cycles that reflector came across due to its movement. The reflector traveled a distance vT, and thus came across vT/λ additional cycles:

$$\text{\# of cycles in } T = f_0 T + \frac{vT}{\lambda}$$

and thus the effective frequency and frequency shift seen by the reflector is:

$$f_{eff} = f_0 + \frac{v}{\lambda}; \qquad f_{shift} = \frac{v}{\lambda} = f_0 \frac{v}{c}$$

For positive v (moving towards the transmitter) the frequency shifts upwards, for negative v downwards.

We now look the complimentary case of a source moving towards the receiver. Here the emitted wavefronts are compressed due to the movement of the source:

The displacement of the source in 1 cycle is v/f_0 and thus the new wavelength is short by that amount:

$$\lambda_{new} = \lambda - \frac{v}{f_0} = \frac{c-v}{f_0}$$

and the effective frequency and frequency shift are:

$$f_{eff} = \frac{c}{\lambda_{new}} = f_0 \frac{c}{c-v}; \qquad f_{shift} = f_0 \frac{c}{c-v} - f_0 = f_0 \frac{v}{c-v}$$

Again, for positive v the frequency shifts upwards, for negative v downwards.

In US, the reflector is both a listener and a source and so we have both shifts occuring. The net frequency shift is:

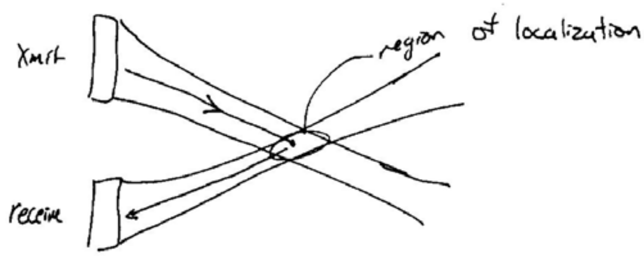
$$\text{net } f_{shift} \approx f_0 \frac{v}{c} + f_0 \frac{v}{c-v} \approx 2 f_0 \frac{v}{c} \quad \text{for } v << c$$

Comments:

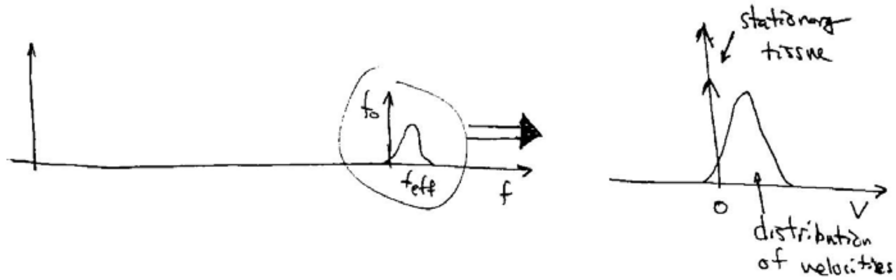
- Observe that the size of the frequency shift is proportional to f_0.
- For physiological blood velocities in the body:
  v = 50 cm/s = 5x10^{-4} mm/μs in major vessels.
  c = 1.5 mm/μs
  relative shift = $2v/c$ = 6.7x10^{-4}
  and for f_0 = 5 MHz, f_{shift} = 3.3 kHz
- All Doppler measurements only account for velocities towards/away from the transducer.

CW Doppler US

One method for doing doppler US measurements is continuous wave (CW) doppler US. In CW imaging, the US signal is transmitted continuously and thus pulsing cannot be used to get depth resolution. This also means that the receiver must be a different transducer than the transmitter. Typically, one uses the intersection of two beams to create an area of localization:



The received signal will have components related to static reflectors at f_0 and perhaps a full spectrum of other signals related to components at different velocities:



In order to distinguish between f_0 and shifted components at f_{eff}, the received signal needs to be observed for a long enough interval, T_{obs}. The frequency resolution will be $1/T_{\text{obs}}$ and the velocity resolution will be:

$$\Delta v = \frac{c}{2T_{obs} f_0}$$

For example, we want velocity resolution of Δv = 5 cm/s = 5x10^{-5}mm/µs and for f_0 = 5 MHz, T_{obs} must be at least 3 ms. This is much longer than the round trip delay for a pulse (which is typically on the order of 100-200 µs).

Comments:

- CW Doppler has excellent velocity resolution and is capable of measuring an entire velocity spectrum from the region of localization
- CW Doppler has very poor range resolution (no pulsing)
- It requires separate transmit and receive transducers
- It is very slow for scanning a large space

Pulsed Doppler

In pulsed Doppler imaging, a train of pulses is sent along a single beam repeatedly. Only one pulse is propagating through the body at any point in time so that maximum rate for pulsing is limited by the roundtrip time for the pulses. For example, if $r_{\max} = 15$ cm, then the pulse repetition time is $T_{PR} = 2r_{\max}/c = 200$ μs. Thus the pulse repetition frequency is $f_{PR} = 5$ kHz. In pulsed Doppler, the pulses are "sampling" the velocity shifts and thus, any frequency shifts larger than $f_{PR}/2$ will be aliased (e.g. to negative frequencies or equivalently, negative velocities).

Another major consideration for pulsed Doppler relates to velocity resolution. Velocities are mapped to frequency shifts and in order to detect small frequency shifts, we need to observe in time for a long period of time, so $T_{obs} \geq \frac{1}{\Delta f_{des}} = \frac{c}{2\Delta v_{des} f_0}$. The total observation time, T_{obs}, must also be long enough so that the fastest moving scatterer interacts with the pulse for that length of time. Since Dopper detects velocities towards and away from the detector only, we need scatterers to remain within the depth resolution element (point spread function) long enough to establish the desired velocity resolution. So, if the resolution element is $\Delta z = cT/2$, then $T_{obs} = \frac{\Delta z}{v_{\max}} = \frac{cT}{2v_{\max}}$. Equating these two expressions for T_{obs}, we get $\frac{T}{f_0} \geq \frac{v_{\max}}{\Delta v}$, that is, the number of cycles for the length of the pulse must be larger than the number of velocity steps that one wishes to resolve.

Therefore, pulsed Dopper US uses much longer pulses than imaging US and thus, has poorer spatial (depth) resolution. Likewise, the frequency f_0 is also limited by the desired depth resolution, and this, of course, affects lateral resolution. Another way to think about this is to recognize that one has to have a narrow spectrum pulse in order to detect small shifts in frequency and long pulses have a narrow spectrum.

- Depth of region of interest, r_{max} determines f_{PR} and thus the frequency shift (velocity) at which aliasing occurs
- Maximum velocity determines frequency f_0
- f_0 and T, determine the maximum frequency shift,
- e.g. for max depth 9.6 cm and maximum velocity f_{PR} = 8 kHz (c = 1.54 mm/ms) maximum shift will be 4 kHz
  for v_{max} = 50 cm/s, this sets $f_{0,max}$ = 6.1 MHz
  for a velocity resolution of 2 cm/s, we then need a pulse length T = 4.1 ms
  this corresponds to a depth resolution of Δz = 6.25 cm.

The above are only approximate relationships. There is also some clever signal processing that can be done to improve the above relationships if we restrict our region of interest to a limited range of depths. For more info on Doppler, please examine professor O'Donnell's Doppler notes at:
http://www.eecs.umich.edu/~odonnel/short_course/chapter3.pdf