Take-Home Exam #1

- You may have up to 48 hours to work on this exam – please enter your name and sign-out and sign-in times on the log sheet.
- You may use your books, your own notes, web resources, computers and calculators.
- You may not seek assistance, share or borrow notes from any current and former students or any other individual (instructors' notes available on the web are OK).
- This exam is governed by the Engineering Honor Code, which requires that you do not seek assistance on this exam and that you report any violations of the Honor Code. (For more info see http://www.engin.umich.edu/org/ehc/index.html).
- On the first page of your solutions, write "I have neither given nor received aid on this exam," and sign your name below it.
- Please read questions carefully and answer all parts.
- Show your work (it's hard to give partial credit without it).
- Include all Matlab scripts with your solutions.
- If parts of the exam are not clear and you cannot find the instructor to ask for clarification, please write your assumptions and proceed with the question. If you cannot solve a part that is needed later in the solution, define a parameter to represent the answer of that part and continue.
- Write legibly (yes I know I don't, but…)
- Please remember to put your name on your exam solutions.
- Good luck!

1. 2D systems (10 pts). For the following 2D continuous domain systems, determine if the system is linear and/or shift invariant. If the system is linear, give its impulse response.
   a. $g(x,y) = S[f(x,y)] = \int_{x-L}^{x+L} \int_{y}^{y+2L} f(x',y')dx'dy'$
   b. $g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x',y')h(ax-x',ay-y')dx'dy'$
   c. $g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x',y')\exp(-i2\pi(x'x+y'y))dx'dy'$
   d. $g(x,y) = |f(x,y)**h(x,y)|$
   e. $g(x,y) = F_{2D}^{-1}\{F_{2D}\{f(x,y)\}\cdot H^2(u,v)\}$

2. Sampling (10 pts). Consider an image, f, with a band-limited spectrum that lies completely in the first quadrant of the continuous Fourier domain (u,v), that is F is non-zero only for $0 \le u < u_m$ and $0 \le v < v_m$. Now, suppose f is sampled at two rates: $f_{s1}(x,y) = f(x,y)comb(2u_m x, 2v_m y)$ and $f_{s2}(x,y) = f(x,y)comb(u_m x, v_m y)$ and the discrete domain images $f_{d1}(n,m)$ and $f_{d2}(n,m)$, respectively, are extracted.
   a. Sketch the Fourier domain for f_{s1} and f_{s2}, identifying where spectrum is non-zero.
   b. Can the original image be reconstructed from f_{d2}? If so, describe the relationship between the discrete samples, $f_{d2}(n,m)$, and $f(x,y)$.
   c. Completely specify $F_{d2}(\omega_x,\omega_y)$ in terms of $F_{d1}(\omega_x,\omega_y)$.

3. Filters (40 pts). In this problem you will design 4 different 7x7 zero-phase, high-pass filters using the methods described below and implement them in Matlab. The desired spatial frequency response is 0 for zero spatial frequency and the 1 for high frequencies. For each filter (a.-d.), calculate the 2D DSFT, $H(\omega_x,\omega_y)$, for at least a 64x64 array and display the real part of H using `imagesc`. Include a `colorbar` and please make sure your axes are labeled and correct.
   a. Starting with a 1-D high-pass filter
   $h_1(n) = [-.0141 \quad -.1111 \quad -.2319 \quad \underline{0.7143} \quad -.2319 \quad -.1111 \quad -.0141]$
   construct a 7x7 separable high-pass filter.
   b. Convert $h_1(n)$ into its corresponding low-pass filter equivalent and construct a 7x7 separable low-pass filter, $h_{\text{lp}}(n,m)$. Now convert $h_{\text{lp}}(n,m)$ into its corresponding high-pass equivalent.
   c. Create a 7x7 high pass filter using the 2-D frequency sampling method. Use this definition for the 2D DSFT domain:
   $$H(\omega) = \begin{cases} 0 & \omega < \pi/7 \\ (\omega - \frac{\pi}{7})\frac{7}{2\pi} & \pi/7 \le \omega \le 3\pi/7 \\ 1 & \omega > 3\pi/7 \end{cases} \quad ,\omega = \sqrt{\omega_x^2 + \omega_y^2}$$
   (Hint: carefully read description of functions `fftshift` and `ifftshift`).
   d. Design an "ideal" (jinc-like) 7x7 filter low pass filter for a cut-off frequency $\omega_c = \frac{2\pi}{7}$. You should use some form of circularly symmetric windowing function. Use this filter as the basis of a high-pass filter.
   e. Comment on differences that you see in these different designs. Which filters are closed to the ideal spectrum (inverse of the ideal low pass in d.)? Are some methods particularly poor and why? Which methods have the narrowest transition band and which have the largest passband ripple? What symmetries exist? Are some computationally better than others?

4. Convolution (40 pts). We wish to filter with the following 5x5 convolution kernel, $h(n,m)$ =

0	-1	-1	-1	0
-1	-1	-1	-1	-1
3	5	<u>5</u>	5	3
-1	-1	-1	-1	-1
0	-1	-1	-1	0

   where the underlined "5" at the center of the matrix is the (0,0) pixel. You will apply this filter to the 58x58 object in the variable `rsqr` produced by e1_p1_template.m. Assume that `rsqr(30,30)` (in Matlab) is the center pixel of the input image. For parts a., b., d., and e., do the following: i) Display the filtered image using `imagesc` and include a `colorbar`, ii) specify the output image size, and iii) specify the Matlab coordinate of the center pixel of the output image.
   a. Implement 2D convolution using `conv2`.
   b. Implement 2D *linear* convolution using 2D FFT's.
   c. Convert `rsqr` into a 1D vector: $r1d(p) = r1d(n+mN) = rsqr(n,m)$ (column-wise stacking in Matlab). Show that a 2D convolution can be implemented by using a 1D convolution of *r1d* with *h1d*, a similar 1D stacking of zero-padded version of *h*. What modifications to the above procedure are necessary to insure a *linear* convolution result? What is the minimum length of *h1d* that yields the linear convolution of part a?
   d. Implement 2D linear convolution using the 1D convolution function `conv`, specicially please implement the ideas of part c.
   e. Implement 2D linear convolution using 1D FFT's. Here, you will implement the 1D convolution of part d. using FFT's.