Take-Home Exam #1

- You may have 25 hours to work on this exam. You must enter your name and sign-out and sign-in times on the log sheet.
- You may use your books, your own notes, web resources, computers and calculators.
- You may not seek assistance, share or borrow notes from any current and former students or any other individual (instructors' notes available on the web are OK).
- This exam is governed by the Engineering Honor Code, which requires that you do not seek assistance on this exam and that you report any violations of the Honor Code. (For more info see http://www.engin.umich.edu/org/ehc/index.html).
- On the first page of your solutions, write "I have neither given nor received aid on this exam," and sign your name below it.
- Show your work (it's hard to give partial credit without it).
- If parts of the exam are not clear and you cannot find the instructor to ask for clarification, please write your assumptions and proceed with the question. If you cannot solve a part that is needed later in the solution, define a parameter to represent the answer of that part and continue.
- Write legibly (yes I know I don't, but…)
- Please remember to put your name on your exam solutions.
- Good luck!

1. In this problem, you will create a Matlab script to investigate methods to suppress particular spectral features in an image. The image sensor has 64 pixels/cm and you've been told that a nearby RF source emitting radiation with a wavelength (1 period) of 0.05 cm (+/- 2%) that generates additive periodic image corruption with the same wavelength. In general, the relative position of the RF source and the image sensor is unknown, so we must assume that the signals can come in from any spatial direction. An example image is available on the course web site in exam_image.mat
   a. What kind of filter is desirable be used to suppress this signal, while preserving most other image features? Describe any desirable symmetries in this filter? What are the critical frequencies (in w_x and w_y) for this filter?
   b. Design a 7x7 filter function to suppress this corruption (there is more than one correct answer here, but please justify your chosen method). Use `imagesc` to show the frequency response. Include a `colorbar` and please make sure your axes are labeled and correct.
   c. Apply this filter using `conv2` and create an image showing the result. Make sure you label the axes (in cm) and make sure that the filter had zero phase.
   d. Create a separable 2D filter in which the 1D component has length 9. Display the frequency response.
   e. Apply this filter by applying the 1D convolution in two directions and create an image showing the result.
   f. Create a 2D frequency domain filter (no restrictions on size) and display this function.
   g. Apply the filter (in frequency domain) and create an image showing the result.
   h. Comment on which methods are best and why. Comment on computational issues in addition to performance issues.

2. You've been hired by NASA to work on data from a defective space telescope. The digital image sensor on this telescope suffered an unfortunate accident with an asteroid, which knocked out the decoding circuitry for ½ of all detector elements. You've discovered that a design engineer anticipated this possibility and created the sensor array in a particular way. The original sampling pattern was rectilinear with spacing T in both x and y directions. The sample locations that remain are those for which (n,m) are both even or for which (n,m) are both odd. The new sampling function can be written as:

$$p(x,y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} d(x-2nT, y-2mT) + \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} d(x-2nT-T, y-2mT-T)$$

   a. Describe the spectrum of object sampled by the original array, given some object spectrum $X(u,v)$ (or $X(w_x, w_y)$, if you prefer). What are the conditions for prevention of aliasing?
   b. Describe the spectrum of object sampled by the defective array. What are the conditions for prevention of aliasing?
   c. Given the alternatives of losing, say, the odd elements in x or the odd elements in y, did the design engineer make a good decision?

3. Consider the following 2D discrete domain filters. For each, determine if the filter is linear and/or shift invariant. If the filter is linear, give its impulse response.
   a. $y(n,m) = \sum_{k=-1}^{1} \sum_{l=-1}^{1} x(k-n, l-m)$
   b. $y(n,m) = \max_{k,l \in (-1,0,1)} x(n+k, m+l)$
   c. $y(n,m) = x(n,m) + x(2n,m) + x(n,2m) + x(2n,2m)$
   d. $y(n,m) = x(n,m) + x(n-2,m) + x(n,m-2) + x(n-2,m-2)$
   e. $y(n,m) = x(m,n)$
   f. $y(n,m) = x^2(n,m)$

4. Sitting in Starbuck's, you overhead a conversation in which someone claimed that you could determine the 2D Fourier transform by using just a single 1D Fourier transform. Flabbergasted, you set out prove or disprove this assertion. You set up the a length NM 1D function $y(p)$ from a NxM 2D function $x(n,m)$:

   $y(p) = y(n+mN) = x(n,m)$ (column-wise stacking, if thinking in Matlab coordinates)

   Letting $Y(q)$ be the 1D DFT of $y(p)$ and $X(k,l)$ be the 2D DFT of $x(n,m)$, you define:

   $X'(k,l) = Y(kM+l) = Y(q)$ (row-wise unstacking).

   a. Determine the relationship between $X'(k,l)$ and $X(k,l)$. Assume the standard definitions of the 1D and 2D DFT's.
   b. For a 32x32 image size and image data defined by $x(n,m) = \text{sinc}(n/4)\text{sinc}(m/2)$, write a Matlab script to compare $X'(k,l)$ and $X(k,l)$. For both of these let the origin of the input and output system at the (17,17) location. Display the images and include real and imaginary parts if appropriate. Please attach the Matlab script and all images.
   c. How close to being correct was the coffee shop assertion?