Take-Home Final Exam

- You may have 72 hours to work on this exam. You must enter your name and sign-out and sign-in times on the log sheet in BIRB 1089. There is a 1 hour grace period.
- You may use your books, your own notes, web resources, computers and calculators.
- You may not seek assistance, share or borrow notes from any current and former students or any other individual (instructors' notes available on the web are OK).
- This exam is governed by the Engineering Honor Code, which requires that you do not seek assistance on this exam and that you report any violations of the Honor Code. (For more info see http://www.engin.umich.edu/org/ehc/index.html).
- On the first page of your solutions, write "I have neither given nor received aid on this exam," and sign your name below it.
- Show your work and hand in all Matlab code.
- If parts of the exam are not clear and you cannot find the instructor to ask for clarification, please write your assumptions and proceed with the question. If you cannot solve a part that is needed later in the solution, define a parameter to represent the answer of that part and continue.
- Write legibly.
- Please remember to put your name on your exam solutions.
- Good luck!

1. [25] Transforms and transform coding. You plan to build and test a transform coder using a Fourier basis set. The problem, of course, is that the complex numbers from the Fourier basis are result in an increase in the number of coefficients for coding of real images. Fortunately, you are aware that there may be some symmetries in the Fourier data that will allow you to reduce the number of coefficient that must be quantized and stored.
   a. We start by solving the 1D case for a length 8 real-valued data vector, $x(n)$, which transforms to real-valued transform coefficients, $X(k)$, also of length 8. Determine the 8 coefficients to be saved and derive the transformation matrix, A. Make whatever modifications are necessary to insure that your transformation is energy preserving. Describe the rows of A (formulas – not numbers) and order them so that the transform coefficients, $X(k)$, will be ordered approximately from largest variance to smallest variance for typical images.
   b. Derive and describe the inverse transformation matrix B.
   c. We now solve the 2D case for an 8x8 real-valued subimage, $x(n,m)$, that transforms to an 8x8 block of real-valued coefficients, $X(k,l)$. Derive and describe this transformation (you do *not* have to explicitly describe a matrix A). Again, please insure that the transformation is energy preserving.
   d. Derive and describe the inverse transformation.
   e. Write a Matlab script to test your solutions to c. and d. The test image will be:

      ```
      im = [1:8]'*[5:12];
      ```

      To insure that the coefficients are real-valued, include the following line:

      ```
      coeffs = real(coeffs);
      ```

      Verify that the transform is energy preserving and invertable. You may use `fft`, `fft2`, `ifft` or `ifft2` in your solution (but you don't have to).

2. [25] Deblurring. You recently got a job with a real estate company to take picture of all of the houses in Ann Arbor. Since you will receive the same amount of money no matter how long it takes, you decided to attach a camera to your car and take picture of homes as you drove by. Unfortunately, most of the pictures are blurred and look like `finalimage2.mat`. Having taken EECS 566, though, you figure that you should be able to undo this blur. First, though, you need to make some assumptions – you assume that the shutter is either fully open or fully closed and that while it is open, your car is moving at a constant velocity, *v*. Unfortunately, you don't know *v*. .
   a. Describe, in 2D, the form of the blur in these images and its frequency response.
   b. Run the script `finalpr2_template.m`. This will produce two plots. Analyze this script and then describe how one might use the data in each of the plots to gather information about the blur function. Derive two estimates of the amount of movement during the exposure, *W*, using only information in these plots. Do not use information from the "reference image" – this is to be used to determine MSE's below.
   c. Use these estimates to implement two deblurring algorithms.
      1) the clipped inverse filter with $g = 3$ (Lim, Eqn 9.50), and
      2) $H(w_x, w_y) = \dfrac{\left(|B(0,0)|^2 + a\right)}{|B(0,0)|^2} \dfrac{B^*(w_x, w_y)}{\left(|B(w_x, w_y)|^2 + a\right)}$, with $a = 0.03$.

      Display the resultant images using `imagesc`. Included with the blurred image, is a reference image of the original unblurred image. Determine the MSE for each method.
   d. You may notice there are substantial edge artifacts. In class, we described several methods for extending images in order to reduce edge effects. Choose a method and explain why that method was chosen. Implement deblurring methods of c. on the extended images, display the images, and determine the MSE.
   e. Which deblurring method has the lowest MSE? From a visual inspection, which appears to be best?
   f. Evaluate the sensitivity of each of the methods to model mismatch. Evaluate the change in MSE with changes in your estimate of blur, *W* (e.g. examine MSE for $W \pm dW$).

3. [10] Vector Quantization (VQ). Consider a WSS image for which you intend to implement 2-length VQ with L^2 reconstruction levels. For which of the following joint probability distributions will VQ be advantageous, in terms of distortion, over scalar quantization with *L* reconstruction levels. For each, describe (qualitatively) how much the improvement from using VQ and why.
   a. $f(x_1, x_2) = \dfrac{1}{2p\sqrt{0.19}} \exp\left\{\dfrac{-1}{0.38}\left[(x_1 - 100)^2 + (x_2 - 100)^2 - 1.8(x_1 - 100)(x_2 - 100)\right]\right\}$.
   b. $f(x_1, x_2) = \dfrac{1}{2ps^2} \exp\left\{-\dfrac{x_1^2 + x_2^2}{2s^2}\right\}$
   c. $f(x_1, x_2) = 4x_1x_2 \quad \text{for} \quad 0 \le x_1, x_2 \le 1$
   d. $f(x_1, x_2) = x_1 + x_2 \quad \text{for} \quad 0 \le x_1, x_2 \le 1$
   e. $f(x_1, x_2) = 0.5*[\mathrm{rect}(x_1\text{-}0.5)\mathrm{rect}(x_2\text{-}0.5) + \mathrm{rect}(x_1\text{-}1.5)\mathrm{rect}(x_2\text{-}1.5)]$

4. [30] Subband-transform coding/Quantization/codewords. In Homework #8, problem 4 we examined the effectiveness of VQ coding for 2x2 blocks of an image. Here we will also operate on 2x2 blocks of an image in a form that could be considered subband coding or very small block transform coding. This problem will again use the house image from homework #7 (`hw7image.mat`). The transformation we will use for a 2x2 subimage, x, is:

$$\begin{bmatrix} C_{LL} \\ C_{HL} \\ C_{LH} \\ C_{HH} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} x(1,1) \\ x(2,1) \\ x(1,2) \\ x(2,2) \end{bmatrix}$$

   a. Implement this transformation on all 2x2 blocks in this image and store the data as subband images, I_{LL}, I_{HL}, I_{LH}, and I_{HH}, each of which is (N/2)x(M/2). Using `imagesc` to present $\begin{bmatrix} I_{LL} & I_{LH} \\ I_{HL} & I_{HH} \end{bmatrix}$. Was this transformation energy preserving?
   b. Implement the inverse transformation to recreate the original image from the subband images.
   c. We plan to assign average bit rates of approximately 5, 1.5, 1.5 and 0 to the subband images I_{LL}, I_{HL}, I_{LH}, and I_{HH}, respectively. Is this a reasonable assignment? Explain.
   d. Quantize I_{LL} using uniform scalar quantization with 5 bits/pixel and make I_{HH} all zeros.
   e. We will quantize I_{HL} and I_{LH} to three levels using a variable length coding. The three reconstruction levels will be $\{+a, 0, -a\}$. I_{HL} and I_{LH} will both use the exact same quantization scheme (same a). Determine the value of a <u>and</u> the decision thresholds that minimize the MSE over I_{HL} and I_{LH}.
   f. Derive a variable length code for the three reconstruction levels of e. Do we achieve our target bit rate of 1.5 bits/pixel?
   g. Estimate the entropy of this image variable resulting from the quantization in e. Describe other coding methods that would allow lower bit rates than those achieved above in f.
   h. Apply 3-level quantization to I_{HL} and I_{LH} and reconstruct the NxM image from the quantized subband images $\begin{bmatrix} \hat{I}_{LL} & \hat{I}_{LH} \\ \hat{I}_{HL} & 0 \end{bmatrix}$. Display the image <u>and</u> the error image using `imagesc`. Determine the average distortion, D. Compare to the distortion from VQ images of equivalent bit rates from HW#8.

5. [10] Systems and transforms. Consider a function $f(x,y) = \mathrm{rect}(x/9)\mathrm{rect}(y/7)$.
   a. Determine its continuous 2D Fourier transform, $F(u,v)$.
   b. Let $f_s(x,y) = f(x,y)\mathrm{comb}(x/2)\mathrm{comb}(y/2)$. Determine its continuous 2D FT, $F_s(u,v)$.
   c. Let $f_d(n,m) = f(2n,2m)$. Determine its 2D Discrete Space FT, $F_d(w_x, w_y)$.
   d. Let $\tilde{f}(n,m)$ be the N,M-periodic extension (or replication) of f_d. Determine its 2D discrete Fourier series, $\tilde{F}(k,l)$.
   e. Let $f_{d2}(n,m) = \begin{cases} \tilde{f}(n,m) & \text{for } 0 \le n < N, 0 \le m < M \\ & \text{otherwise} \end{cases}$. Determine its 2D DFT, $F_{d2}(k,l)$.