

**Homework #1**

Due Date: Jan. 17, 2002

1. [10] For the following systems, determine if the system is linear and if so determine the PSF. Also, determine if the system is space-invariant. Let  $a, b$  be non-zero, real numbers.
  - a.  $g(x,y) = S[f(x,y)] = f(ax, ay)$
  - b.  $g(x,y) = S[f(x,y)] = f(x-a, y-b)$
  - c.  $g(x,y) = S[f(x,y)] = |f(x,y)|$
2. [5] State and prove the condition on  $h(x,y)$  in order for a linear space invariant system to be rotationally invariant.
3. [10] Show or prove the following properties of 2D convolution.
  - a. Shift property:  $f(x, y) ** \mathbf{d}(x - x', y - y') = f(x - x', y - y')$
  - b. Shift invariance:  $f(x, y) ** h(x, y) = g(x, y)$  implies that  $f(x - x', y - y') ** h(x - x', y - y') = g(x - x', y - y')$
  - c. Circular symmetry: If  $f(x, y)$  and  $h(x, y)$  are circularly symmetric, then  $f(x, y) ** h(x, y)$  is also circularly symmetric.
4. [5] Prove that  $f(x, y) ** \mathbf{d}(x - a, y - b) = f(x - a, y - b)$ .
5. [10] Find the 2D Fourier transforms of:
  - a.  $\text{sinc}(ax-b)$
  - b.  $\text{sinc}(x-a)\text{rect}(by)$
  - c.  $g_r(ar)$  given that  $F\{g_r(r)\} = G(\mathbf{r})$ .
6. [10] Determine the spatial resolution using, i) the Rayleigh criterion, ii) the Sparrow criterion, and iii) FWHM, for the following functions. Matlab's `fzero` may be useful here.
  - a.  $h(x) = \text{sinc}(x)$
  - b.  $h(x, y) = \exp(-\mathbf{p}(x^2 + y^2))$

7. [100] Consider an imaging system with frequency response:

$$H(\mathbf{r}) = \exp(-\mathbf{p}(\mathbf{r}/16)^2) - \exp(-\mathbf{p}(\mathbf{r}/4)^2).$$

We would like to determine what the output image would be if the input image were

$$f(x, y) = \text{rect}(x/3, 3y) + \text{rect}(2x, y)$$

One could solve this problem using convolution, but an easier way is to use MATLAB's `fft2` command to compute the output image  $g(x, y)$ . Caution: you must be very careful with `fftshift` and your sample locations to get a correct answer. To work on this problem, you may wish to download the template file `h1template.m` from the web site.

- a. Display the real and the imaginary parts of your result as two distinct images using `subplot`. Display a `colorbar` to give the amplitude scale.
- b. Show  $|F(u, v)|$  using `subplot` and `colorbar`.
- c. Is your resulting  $g(x, y)$  exact or approximate at the sample locations? If not, why?