EECS 556, Imaging Processing, Douglas C. Noll 3/12/2002

Homework #6

Due Date: Mar. 18, 2002

1. [30] Consider the following two 1D interpolation (cubic and quadratic) kernels:

$$h_c(x) = \begin{cases} 1 - 2.5x^2 + |x|^3, & |x| \le 1 \\ 2 - 4|x| + 2.5x^2 - 0.5|x|^3, & 1 < |x| \le 2 \\ 0 & \textit{otherwise} \end{cases}$$

$$h_q(x) = \begin{cases} 1 - 2x^2, & |x| \le 0.5 \\ 1.5 - 2.5|x| + x^2, & 0.5 < |x| \le 1.5 \\ 0 & \textit{otherwise} \end{cases}$$

   a. Plot each kernel and determine and plot the 1st and 2nd derivatives for $x \ge 0$.
   b. Set `x = linspace(-10,10,1024)` and use `fft` to determine their spectra. Normalize each so that the maximum value is 1. Plot using `semilogy`. Be sure the frequency axis is correct.
   c. Evaluate these methods with respect to criteria discussed in class ($h(0) = 1$, $h(z \neq 0) = 0$, continuity of derivatives, rect-like spectrum, compactness for computational reasons).

2. [10] Show that the "filtered Bernoulli point process" signal $z(n;m)$, presented in class and near the end of Fessler's Notes 6.14, is WSS, and derive its autocorrelation function.

3. [10 each] Lim, Problems 9.11, 9.16

4. [90] Wiener filters:
   a. From the web page, download the MATLAB m-file `hw6template.m`. Run the file and look at the image displayed. Analyze the code and determine the *theoretical* autocorrelation function of the random process $s(n;m)$ that is first generated by this code.
   b. Determine analytically the power spectrum of that random process.
   c. Use MATLAB to compute *empirically* the autocorrelation function of $s(n;m)$ using MATLAB's `xcorr` function. Display both the empirical estimate and the theoretical autocorrelation function.
   d. Examine the code to understand the signal $y(n;m)$ generated therein. Determine analytically the frequency response $H(w_x, w_y)$ of the Wiener filter that is the minimum MSE LSI estimator of $s(n;m)$ given $y(n;m)$.
   e. Implement the Wiener filter using FFT's and display the filtered image $\hat{s}(n,m)$. Hint: since power spectra are real, your Wiener filter should be real too.
   f. Download the m-file `anisodiff.m`, which is a non-linear adaptive filter (I got this file from: http://www.cs.uwa.edu.au/~pk/Research/MatlabFns/, a nice collection of image processing Matlab functions). Apply this to $y(n;m)$ using the command `shatad = anisodiff(yy,4,10,.2,1);`. Display your result.
   g. Compare the Wiener filter and adaptive filter both qualitatively and using the mean square error (MSE) measure. In terms of the MSE, do the results you get contradict the optimality of the Wiener filter? Explain.