Homework #9

This is for practice only – do not hand in.

1. Energy compaction in transform coding. Download the template `hw9_template.m` for use in this problem. This problem will again use the house image from homework #7. We will examine three different transforms: the DCT, the Hadamard and Haar transforms and we will use 8x8 blocks. The size 8 1D Haar transform matrix is:

$$Haar_8 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 \end{bmatrix}$$

   The 1D Hadamard transform matrix can be determined from the Matlab `hadamard` command. The 2D DCT can be derived from Matlab's `dct` command. I would avoid using the `dct` function we created in homework #3 as that one was not an energy preserving transformation.

   a. Implement 8x8 transform for DCT, Hadamard, and Haar. Verify that each is energy preserving using a sample 8x8 image (sum squared energy in transform domain will be the same as sub squared energy in the image – or that $H*H' = I$)
   b. Take the 8x8 transform for all 8x8 blocks in the image and store the transform coefficients in a 64x5400 array.
   c. Determine the variance of each of the coefficients and sort by magnitude.
   d. Plot the variances in increasing or decreasing order for each method. Use a log scale (`semilogy`) and put all three methods on one plot.
   e. Create a plot of the cumulative variance in the *n* highest coefficients for each method. Use a linear scale and put all three methods on one plot. Which method has the best energy compaction.
   f. Determine the number of bits that should be assigned to each coefficient for an average bit rate of 1 bit/pixel. Round to the nearest integer and disallow the assignment of negative bits/pixel. Which transform method results in the most coefficients coded with zero bits/pixel.