A Temporal Model for Task-based Functional MRI Reconstruction

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Introduction

- Goal: better identify task-activated brain regions in **task-based fMRI**.
- Model: to separate **task-correlated** signal from **non-task** background.
- Novelty: use *a priori* knowledge of **activation waveform shape**, and **temporal smoothness** assumption of background.
- Merit: advance model-based **reconstruction** from undersampled k-space.

Problem Formulation

Reconstruct MR image series from undersampled k-space data:

$$\underset{X}{\operatorname{argmin}} \frac{1}{2}\|\mathbf{E}X - d\|_2^2 + \lambda R(X)$$

$\mathbf{E} : \mathbb{C}^{N_v \times N_t} \to \mathbb{C}^{N_s}$	data acquisition operator (where N_v = number of voxels, N_t = number of time frames, N_s = number of k-space samples)
$X \in \mathbb{C}^{N_v \times N_t}$	desired image series
$d \in \mathbb{C}^{N_s}$	undersampled k-space data
$R(\cdot)$	regularizer with parameter λ

Existing Models

- Low-Rank Plus Sparse Decomposition (L+S) [1], [2]

$$\underset{L,S}{\operatorname{argmin}} \frac{1}{2}\|\mathbf{E}(L+S) - d\|_2^2 + \lambda_L\|L\|_* + \lambda_S\|\mathbf{T}S\|_1$$

$L \in \mathbb{C}^{N_v \times N_t}$	**non-task** background
$S \in \mathbb{C}^{N_v \times N_t}$	pseudo-periodic **task** signal
$\mathbf{T} : \mathbb{C}^{N_v \times N_t} \to \mathbb{C}^{N_v N_t}$	temporal Fourier transform operator

- Low-Rank Plus Task-Based Decomposition (L+UV) [3]

$$\underset{L,U}{\operatorname{argmin}} \frac{1}{2}\|\mathbf{E}(L+UV) - d\|_2^2 + \lambda_L\|L\|_*$$

$L \in \mathbb{C}^{N_v \times N_t}$	**non-task** background
$U \in \mathbb{C}^{N_v \times N_r}$	estimated **task** spatial map
$V \in \mathbb{C}^{N_r \times N_t}$	temporal basis with activation waveform

Proposed Model

Smooth Background Plus Spatial-Temporal Decomposition (B+UV)

$$\underset{B,U}{\operatorname{argmin}} \frac{1}{2}\|\mathbf{E}(B+UV) - d\|_2^2 + \lambda_B\|\mathbf{D}B\|_2^2 \qquad (1)$$

$B \in \mathbb{C}^{N_v \times N_t}$	temporally smooth **non-task** background
$U \in \mathbb{C}^{N_v \times N_r}$	estimated **task** spatial map
$V \in \mathbb{C}^{N_r \times N_t}$	temporal basis with activation waveform and scanner drift
$\mathbf{D} : \mathbb{C}^{N_v \times N_t} \to \mathbb{C}^{N_v N_t}$	temporal finite difference operator

Optimization Algorithm

- Compatibility of vectorization with Kronecker product: vec$(UV) = (V^\top \otimes I)vec(U)$
- Write $\mathbf{E}(UV) = \mathbf{E}_v U$, $\widetilde{\mathbf{E}} = [\mathbf{E}\ \mathbf{E}_v]$, $\widetilde{X} = [B\ U]$, $\widetilde{\mathbf{D}} = [\mathbf{D}\ \mathbf{0}]$, then (1) becomes

$$\min_{\widetilde{X}} \frac{1}{2}\|\widetilde{\mathbf{E}}\widetilde{X} - d\|_2^2 + \frac{\lambda_B}{2}\|\widetilde{\mathbf{D}}\widetilde{X}\|_2^2$$

- Practical implementation: conjugate gradient (CG) method

Advantage over existing models:

- L+S: incoherence between L and S might not apply, and temporal Fourier sparsity assumption of S might not capture activation
- L+UV: both terms are low rank, might not separate signal from background
- B+UV: incoherence between smooth background signal B and task UV

Results

Simulated task: resting-state fMRI with 2 activated Gaussian regions of interest (ROI) in k-space, 32 coils, $N_v = 100 \times 100$, $N_t = 300$, $4\times$ undersampling

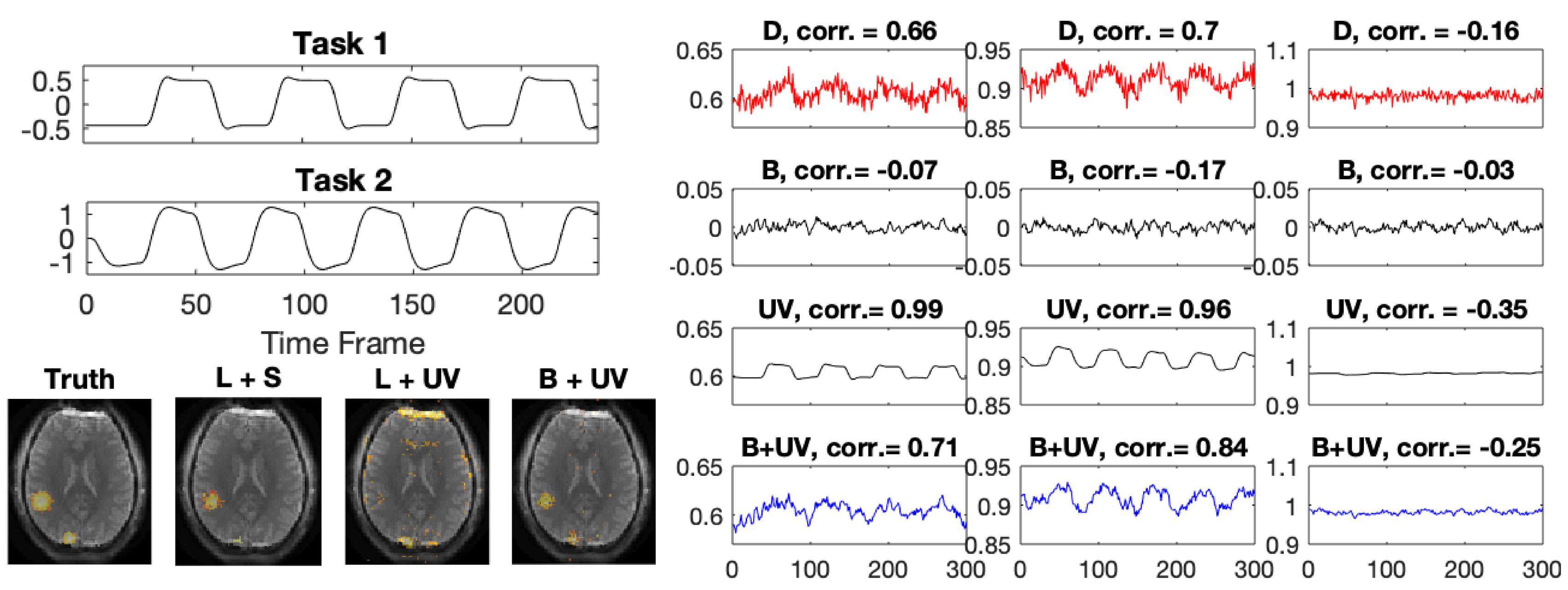

Figure 1: Left: task waveforms and activation maps by all reconstruction results. Right: B+UV timeseries of two task-activated voxels and a non-task voxel.

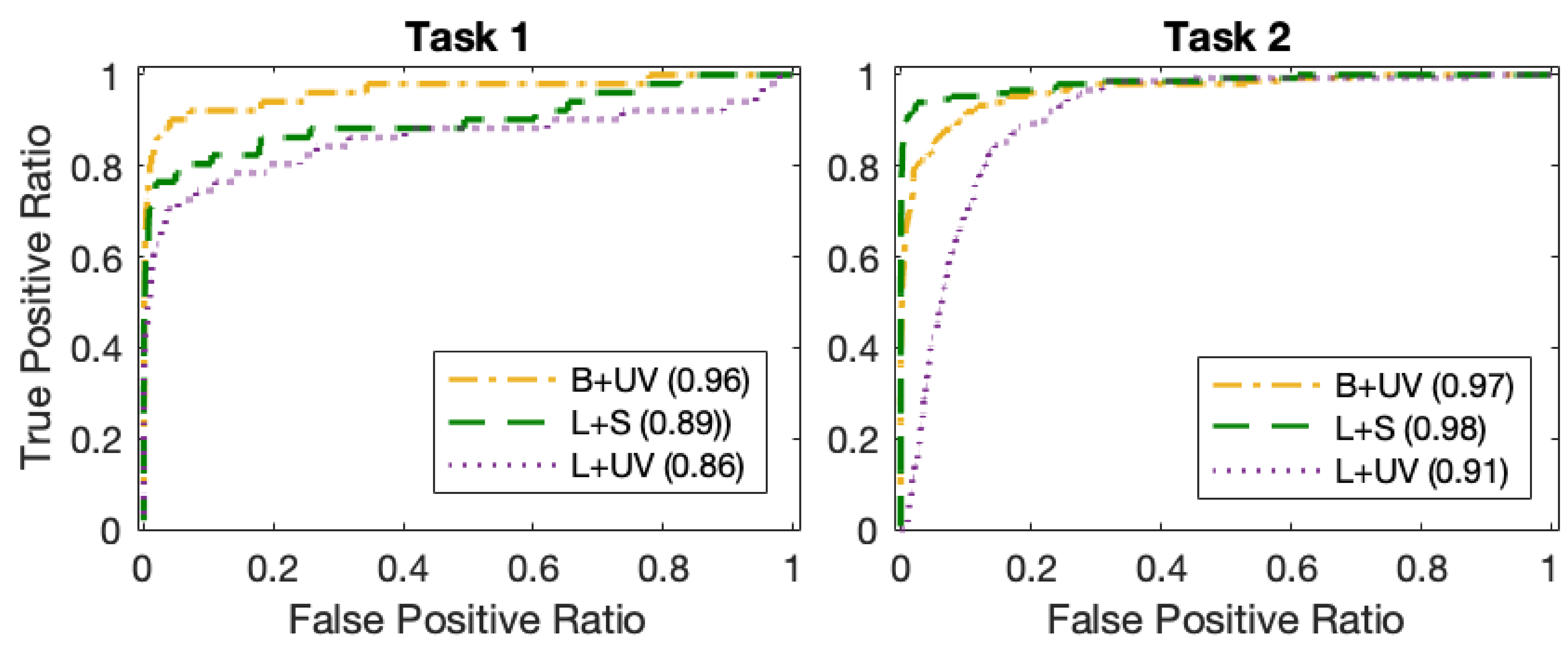

Figure 2: Receiver operating characteristic (ROC) curves across activation thresholds with Area Under Curve (AUC).

Finger Tapping Task: 3D task fMRI, 32 coils, $N_v = 72 \times 48 \times 10$, $N_t = 235$, $4\times$ undersampling

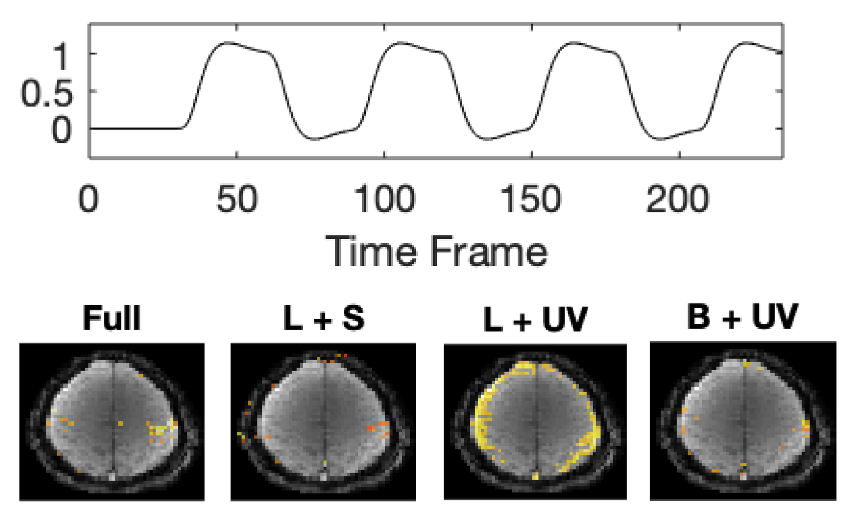

Figure 3: Task waveform and activation maps.

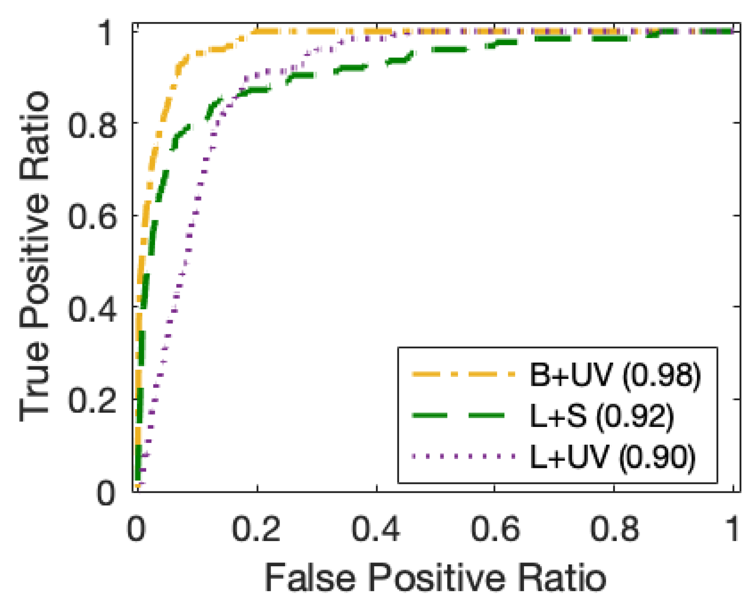

Figure 4: ROC curves with AUC values.

Conclusion

- Proposed B+UV model improves **activation detection** compared with existing fMRI models, as seen by higher AUC values.
- B+UV components **separate** task signal and non-task background.
- Solving B+UV is **computationally advantageous** with simple CG updates.

Acknowledgement

This work is supported in part by NIH grant R01 EB023618.
We thank Amos Cao and Michelle Karker at the University of Michigan for the fMRI datasets.

References

[1] R. Otazo, A. Franco, J. Chen, C. Marmar, and F. Boada, "Low-rank plus sparse (L+S) decomposition for separation of subsampled physiological noise in fMRI," *OHBM*, vol. 1690, p. 2015, 2015.

[2] V. Singh, A. H. Tewfik, and D. B. Ress, "Under-sampled functional MRI using low-rank plus sparse matrix decomposition," *IEEE ICASSP*, pp. 897–901, 2015.

[3] M. Chiew, N. N. Graedel, and K. L. Miller, "Recovering task fMRI signals from highly under-sampled data with low-rank and temporal subspace constraints," *NeuroImage*, vol. 174, pp. 97–110, 2018.