Remarks on the Problem of Logical Omniscience

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These comments on "Logical omniscience and the sense of epistemic modals," by Craige Roberts and Stewart Shapiro, were presented at a workshop on Modality and Natural Language Metaphysics at the Ohio State University on March 23, 2016.

In running through these comments,
I'll try to be brisk.

R&S's Study Divides into Two Parts

- *Part 1:* Logical omniscience and epistemic modals.

- *Part 2:* Practical reasoning.

- I'm going to concentrate on Part 1. Practical reasoning brings in problems of limited rationality in autonomous agents that are far more general, and it would take too long even to begin to do justice to it.

I. The Problem

Agreement about the Problem

- It's good to begin with mathematical examples, ...

- because they make it clear that a solution in terms of possible worlds semantics is out of the question.

- This doesn't mean that we should give up possible worlds semantics for modals and propositional attitudes.

- As usual in science, we retain theories with known limitations until we have something better to replace them with.

What's the Problem?

- I agree with R&S that a very good formulation is this:

  What can be readily inferred from a given evidential basis?

- The problem, then, is how to formalize *evidential basis* and *inferential difficulty*.

Criteria for a Solution

- I agree with R&S's criteria (evidential base, distinctions finer than logical equivalence, robust relations to attitudes like belief, graspability, and publicity), ...

- as far as epistemic modals go.

- But I want to point out that the problem is more general, and there are other, more urgent applications at stake that will probably bring in other criteria.

Scope and Difficulty

- The scope and difficulty of these problems is often not appreciated.

- Personally, I don't think we're close to an adequate solution.

- Probably the best we can do is to make the problem as clear as possible, ...

- in the hope that future generations may find a solution.

What Theories Are Available?

- The best current source on this topic is

  Joseph Y. Halpern and Riccardo Pucella,
  "Dealing with Logical Omniscience:
  Expressiveness and Pragmatics" *Artificial
  Intelligence* 175.1 (2011), pp. 220–235.

- They discuss four theoretical approaches: syntactic,
  theories that incorporate awareness, algorithmic
  knowledge, and impossible worlds.

- The pragmatic side has to do with how to develop
  useful applications of these theories.

II. A Couple of Miscellaneous Remarks

Mathematics or Psychology?

- I think it's wrong to think of Barbara Partee's title as a dilemma.

- Psychology, like computer science, has to deal with its subject matter at different theoretical levels.

- Many psychological theories involve mathematics.

- But in particular, what Allen Newell called *the knowledge level* uses logic and related theories to specify the problem that is being solved.

- You can often get quite far in psychology by assuming that cognitive mechanisms are producing optimal or near-optimal solutions.

- For instance, Bayesian learning theory is widely used this way in contemporary psychology.

- Of course, there are limits to how far you can take this idea—it obviously is going to play out in domains where the reasoning problems are intractible.

- Transcending those limits is a psychological version of the problem of logical omniscience.

Deduction Versus Abduction

- Proving mathematical theorems is a deductive problem.

- Inferring plausible hypotheses from evidence (as in crime detection examples) is an abductive problem.

- I'm quite sure that logical omniscience is a problem in both cases, ...

- but there may be important differences in how the problem plays out in deduction and abduction, so we shouldn't ignore the fact that these reasoning processes are not the same.

III. The Generality of the Problem

- Epistemic modals are an instance of the problem, but there are many other instances, and to arrive at a solution we have to be aware of this.

- The problem arises in:

  1. Computer science (and especially in cryptography).
  2. Microeconomics.
  3. Psychology.

- And it isn't restricted to epistemic modals.

Computer Science

- Informally, you want to say that standard encryption algorithms guarantee that a document can't be accessed except by authorized agents.

- This uses a syntactic account of information—accessing a document amounts to being able to reproduce an exact copy.

- And it assumes that any agent that attempts to decrypt the document will use a specific algorithm to factor the product of two prime numbers.

- That enables you to calculate how long it will take to do the factorization, and by making the numbers large enough you can make this time long enough for the calculation to be infeasible. (Longer than the expected life of the universe, if you like.)

This isn't Anything Like
a General Theory of Information Availability

- Imagine a large community of agents sharing information.

- Some subcommunities would like to keep some things private.

- This means, among other things, that the information—e.g. the personal data associated with a patient record—can't easily be pieced together from public information—say, from sanitized patient records and from information easily available on the web.

- Finding a theory of this would require a theory of information that isn't merely syntactic, •••

- and that would allow you to estimate the difficulty of reaching certain conclusions from available evidence.

- This looks a lot like R&S's account of what a theory of epistemic modals should deliver.

Digression

- Modalities involving ability exhibit the same problematic features as epistemic modals.

- Suppose the algorithm for factoring $p_1 \times p_2$ is to randomly select two prime numbers less than $p_1 \times p_2$.

- This algorithm "can" perform the factorization in one step.

- Finding a meaning for 'can' that would make sense of the more general claim that however an agent tries, it can't factor the product of two large primes, raises exactly the same problems that R&S raise for epistemic modals.

- A more familiar example is the common sense claim that I can't log into your email account because I don't know the password.

Microeconomics

- This is a more complicated situation, if you consider all the varieties of practical reasoning, •••

- but you run into problems very similar to the ones we've been considering in the cases to which game theory and decision theory can be readily applied.

- When there is complete information, and probabilities aren't needed, we still have to reckon with the possibility that an agent may not be able to readily infer conclusions from the available evidence that may be crucial for the decision at hand.

- In cases where we need to work with probabilities, we need to generalize the qualitative problems of logical omniscience to probability functions.

- E.g., we need to be able to deal with cases where joint probabilites aren't known or are incorrectly calculated.

IV. Quick Conclusion

The Problem is General

- I hope that I've persuaded you that it shows up in any areas of cognitive science where something like "rationality" is at stake.

- And that a solution to it could have practical applications of great importance.

- *Conclusion:* An adequate solution to the problem must be general.

There Are No Adequate Solutions

- R&S point out problems with the "impossible worlds" approach.

- I claim you can make equally convincing cases for the other approaches that have been suggested.

- In particular, if a solution suggests specific representations for propositions, you must require that you can show in some applications at least that these representations help to model the reasoning in a plausible and useful way.

The Problem is Challenging

- This is not the sort of problem you'd want to suggest to a graduate student looking for a dissertation topic.

- It's not the sort of problem that is likely to be solved by a large, well funded research team, because that would require a way of dividing the problem up into manageable subproblems.

- It looks as if we can't hope for progress without a fundamental theoretical breakthrough of some sort.

- *My conclusion:* Until that happens, the best we can do is to state the problem as clearly and as generally as possible, and to draw attention to its importance.