Truth After the Fact in Indeterminist Tense Logic

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Date of this version: March 24, 2007

1. Introduction

Branching conceptions of time invoke temporal models in which the same synchronic state is associated with multiple future outcomes. *Indeterminist* conceptions of time add to this picture the idea that a sentence can be true at a moment with alternative futures only if its truth depends only on the history of things up to and including that moment. In particular, consider an atomic formula p, and suppose that the truth of p at a moment depends only on the local state associated with that moment—p makes no claims regarding the future. On an indeterminist conception, the truth of Fp[1] at m should not depend on any assumptions concerning how things turn out after m.

Indeterminist approaches to time, change, knowledge, causality and freedom are challenging; it can be difficult to turn them into theories that are at once coherent, intuitively satisfactory, and reasonably detailed. The problem of how to construct a plausible indeterminist account of satisfaction for tense logic in branching time models is a (relatively small) part of this general problem.[2] It turns out to be surprisingly difficult to accommodate intuitive validities like $F\top \rightarrow (Fp \vee F\neg p)$ in indeterminist branching time models. (For details, see [Thomason, 1970], [Belnap, Jr. *et al.*, 2001][Chapter II], [MacFarlane, 2002].)

The solution that I proposed in [Thomason, 1970] uses van Fraassen's method of supervaluations,[3] and so divides the model theoretic account of satisfaction into two parts.

(1) First, $\mathcal{M}, m, h \models A$, "Model $\mathcal{M}$ satisfies A at moment m and supposing history h to represent the ensuing future," is defined by a recursion of the sort that is routinely used in modal logic. (In particular, satisfaction of a future-tense formula ϕ depends on the satisfaction of ϕ at moments subsequent to m on h; $\mathcal{M}, m, h \models F\phi$ if and only if $\mathcal{M}, m', h \models \phi$ for some m' on h such that $m < m'$.) This secures the validity of $F\top \rightarrow (Fp \vee F\neg p)$. But this account of satisfaction is not indeterminist, because $\mathcal{M}, m, h \models Fp$ may hold for some h and not for others, and so the truth at m of some formulas will depend essentially on what is supposed to happen after m.[4]

(2) Second, satisfaction *simpliciter* at a moment m is defined by letting $\mathcal{M}, m \models \phi$ if and only if $\mathcal{M}, m, h \models \phi$ for all h passing through m. This exhibits the characteristic supervaluationist combination of Excluded Middle without Bivalence—it validates $\phi \vee \neg\phi$ without in

[1] Here, F is the future tense operator. In Arthur Prior's commonly used notation for tense operators, F is future, P is past, G is the dual $\neg F \neg$ of F, and H is the dual $\neg P \neg$ of P. Below, in Section 2, I will advocate and adopt a different notation.

[2] Arthur Prior gets the credit for framing the technical problem, in [Prior, 1967].

[3] See [van Fraassen, 1969].

[4] Prior calls this the "Ockhamist" theory of satisfaction, probably supposing that Ockham would think of the truth-ensuring h as the unique future course of events ensured by the will of God.

general postulating that either $\mathcal{M}, m \models \phi$ or $\mathcal{M}, m \models \neg\phi$. And $F\top \to (Fp \vee F\neg p)$ is valid, while there are models $\mathcal{M}$ such that $\mathcal{M}, m \models F\top$, but $\mathcal{M}, m \not\models Fp$ and $\mathcal{M}, m \not\models F\neg p$ for some moment m in the model's frame. For details, see [Thomason, 1970], [Belnap, Jr. *et al.*, 2001][Chapter II], [MacFarlane, 2002].

The details of the formal theory are presented in the following section. The presentation reproduces the theory of [Thomason, 1970], but with changes to bring notation into line with the contemporary approach to the model theory of modal logic, as explained, for instance, in [Blackburn *et al.*, 2001].

2. Satisfaction and validity in indeterminist tense logic

The language of the tense logics of interest to us are the closures of the propositional constant $\top$ and a set $\mathcal{P}$ of propositional atoms under boolean operators, a fixed set $\{\langle\mathrm{P}\rangle, \langle\mathrm{F}\rangle, [\mathrm{P}], [\mathrm{F}]\}$ of tense operators, and a set of modal operators.[5]

I will assume that p and q are members of V. The boolean operators include $\neg$, $\wedge$, $\vee$ and $\to$. The modal operators will be a subset of $\{[\mathrm{H}], [\mathrm{T}]\}$; this provides, of course, for four languages, one for each subset. The three most important of these for the purposes of this paper are $\mathcal{L}_0$, whose set of modal operators is empty, $\mathcal{L}_1$, whose set of modal operators is $\{[\mathrm{H}]\}$, and $\mathcal{L}_2$, whose set of modal operators is $\{[\mathrm{H}], [\mathrm{T}]\}$.

Definition 2.1. Frames, histories, valuations, models.

A *frame* for indeterministic logic is a pair $\mathcal{F} = \langle M, \preceq\rangle$ consisting of a nonempty set M (the set of moments) and a transitive, antireflexive relation $\prec$ over M such that if $m_1, m_1 \prec m$, then $m_1 \prec m_2$ or $m_2 \prec m_1$.

Let $\mathcal{F} = \langle M, \preceq\rangle$ be a frame. A *history* h on $\mathcal{F}$ is a maximal $\prec$ chain on $\mathcal{F}$. That is, (1) $h \subseteq M$, (2) for all $m, m \in h$, either $m \prec m'$ or $m' \prec m$, and (3) for all chains h' on $\mathcal{F}$, if $h \subseteq h'$ then $h = h'$. When $m \in h$, we say that m is *on* h, or that h *passes through* m or *is through* h. $\mathcal{H}_m$ is the set of histories passing through m.

A *valuation* of a set $\mathcal{P}$ of propositional atoms on a frame $\mathcal{F} = \langle M, \preceq\rangle$ is a function V from $\mathcal{P}$ to the power set of M.[6]

A *model* on a set $\mathcal{P}$ of propositional atoms is a pair $\langle \mathcal{F}, V\rangle$, where $\mathcal{F}$ is a frame and V is a valuation of $\mathcal{P}$ on $\mathcal{F}$.

The definition of satisfaction relative to a postulated history then proceeds as follows, for the *base* tense language $\mathcal{L}_1$ with historical necessity $[\mathrm{P}]$ as its only modal operator.

[5] When working with languages like this, which invoke many modalities, it is useful to have a notation for modalities that (1) is flexible in providing easily recognizable names for complex modalities and (2) represents duality explicitly, like the traditional We can achieve both desiderata by splitting '$\Box$' and '$\Diamond$' into halves; this gives us square and angle brackets '[ ]' and '$\langle\ \rangle$'. between the brackets we can then introduce a descriptor for the modality between the brackets to produce nomenclature for arbitrary modalities. With 'F' for "future-oriented", 'P' for "past-oriented", and 'H' for "historical," we then have '$[F]$' and '$\langle F\rangle$' for future-oriented necessity and possibility (Prior's G and F), '$[P]$' and '$\langle P\rangle$' for past-oriented necessity and possibility (Prior's H and P), and '$[H]$' for historical necessity.

[6] This characterization of models validates $\phi \leftrightarrow [\mathrm{H}]\phi$ for $\phi \in \mathcal{P}$. Treating propositional atoms in this way as temporally local (i.e., as dependent only on the moment of evaluation, not the history) simplifies the logic, but can be avoided by allowing valuations to be functions from $\mathcal{P}$ to $\{\langle m, h\rangle \,/\, h \in \mathcal{H}_m\}$.

Definition 2.2. $\mathcal{M}, m, h \models \phi$, for formulas ϕ of $\mathcal{L}_1$.

1. **Basis:** If $\phi \in \mathcal{P}$ then $\mathcal{M}, m, h \models \phi$ iff $m \in V(\phi)$.
2. **Booleans:** Boolean conditions are routine.
3. **Past:** $\mathcal{M}, m, h \models \langle P \rangle \phi$ iff for some $m' \prec m$, $\mathcal{M}, m', h \models \phi$.
   $\mathcal{M}, m, h \models [P]\phi$ iff for all $m' \prec m$, $\mathcal{M}, m', h \models \phi$.
4. **Future:** $\mathcal{M}, m, h \models \langle F \rangle \phi$ iff for some m', $m \prec m'$ and $m' \in h$, $\mathcal{M}, m', h \models \phi$.
   $\mathcal{M}, m, h \models [F]\phi$ iff for all m', $m' \prec m$ and $m' \in h$, $\mathcal{M}, m', h \models \phi$.
5. **Historical Necessity:** $\mathcal{M}, m, h \models [H]\phi$ iff for all h' passing through m, $\mathcal{M}, m, h' \models \phi$.

Base Semantic Rules

The Base Semantic Rules provide a definition of satisfaction in a model at a moment, relative to a postulated history. The following definition of satisfaction *simpliciter* follows the usual supervaluational policy.

Definition 2.3. $\mathcal{M}, m \models \phi$, for formulas ϕ of $\mathcal{L}_1$.
$\mathcal{M}, m \models \phi$ iff for all $h \in \mathcal{H}_m$, $\mathcal{M}, m, h \models \phi$.

Finally, validity and implication are defined in terms of satisfaction *simpliciter*.

Definition 2.4.[7] $\Gamma \Vdash \phi$, $\Vdash \phi$, for formulas of $\mathcal{L}_1$.
$\Gamma \Vdash \phi$ iff for all frames $\mathcal{F} = \langle M, \prec \rangle$, for all models $\mathcal{M}$ on $\mathcal{F}$ and all $m \in M$, we have $\mathcal{M}, m \models \phi$.
$\Vdash \phi$ (ϕ is *valid*) iff $\emptyset \Vdash \phi$.

As usual in supervaluational semantics, we have a distinction between implication and the validity of the conditional. For instance, $\nVdash \langle F \rangle p \rightarrow [H]\langle F \rangle p$ (the formula $\langle F \rangle p \rightarrow [H]\langle F \rangle p$ is not valid), whereas $\{\langle F \rangle p\} \Vdash [H]\langle F \rangle p$ ($\langle F \rangle p$ implies $[H]\langle F \rangle p$).

3. Object-level truth in indeterminist tense logic

So far, unless the appeal to supervaluations is considered to be problematic, the interpretation of indeterminist tense logic is straightforward. The patterns of validity in this logic are familiar and plausible; validity is, in fact, equivalent to validity in Prior's Ockhamist system, and the $\mathcal{L}_0$ fragment is equivalent to standard tense logic.

However, when a modal operator [T] for truth is added to the mix, things do become perplexing. In [Thomason, 1970], I introduce a truth operator, but the semantics that I gave for [T] doesn't match the supervaluational treatment of truth used in the metalanguage. The semantic rule that I gave for T in [Thomason, 1970] is:

Rule 3.1.
$m, h \models [T]\phi$ iff $m, h \models \phi$.

[7] In using '$\Vdash$' for implication, I differ from the more usual notation seen in [Blackburn *et al.*, 2001]. I prefer not to use the satisfaction symbol '$\models$' for implication and validity.

And the corresponding definitions of satisfaction, implication and validity are framed as follows.

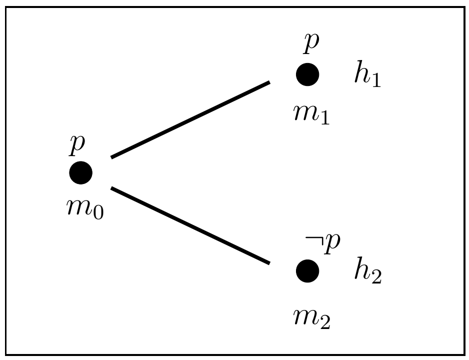
Definition 3.1. Satisfaction$_1$, $\Vdash_1$, valid$_1$.

$\mathcal{M}, m, h \models_1 \phi$ is characterized for $\mathcal{L}_1$ by the Base Semantic Rules together with Semantic Rule 3.1. And as usual, $\mathcal{M}, m \models_1 \phi$ iff $\mathcal{M}, m, h \models_1 \phi$ for all $h \in \mathcal{H}_m$.

$\Gamma \Vdash_1 \phi$ iff for all frames $\mathcal{F} = \langle M, \prec \rangle$, $\mathcal{M}, m \models_1 \phi$ for all models $\mathcal{M}$ on $\mathcal{F}$ and all $m \in M$.

$\Vdash_1 \phi$ (ϕ is *valid*$_1$) iff $\emptyset \Vdash_1 \phi$.

This interpretation of [T] makes $[\mathrm{T}]\phi$ and ϕ equivalent in the strongest sense: for all models $\mathcal{M}$, $\mathcal{M}, m, h \models_1 [\mathrm{T}]\phi$ iff $\mathcal{M}, m, h \models_1 \phi$, for all moments m and histories h of $\mathcal{M}$'s frame. Consider model $\mathcal{M}_1$, for instance. Here, $\mathcal{M}_1, m_0, h_1 \models_1 [\mathrm{T}]\langle \mathrm{F} \rangle p$ because $\mathcal{M}_1, m_0, h_1 \models_1 \langle \mathrm{F} \rangle p$ and $\mathcal{M}_1, m_0, h_2 \models_1 \neg[\mathrm{T}]\langle \mathrm{F} \rangle p$ because $\mathcal{M}_1, m_0, h_2 \models_1 \neg\langle \mathrm{F} \rangle p$.



Model $\mathcal{M}_1$

But then, $\mathcal{M}_1, m \not\models_1 \neg[\mathrm{T}]\langle \mathrm{F} \rangle p$. However, our supervaluational approach to truth at m_0 *falsifies* the claim that $\langle \mathrm{F} \rangle p$ is true at m_0 in $\mathcal{M}_1$. We want, in particular, to say that $\langle \mathrm{F} \rangle p$ is neither true nor false at m_0, so we want to deny that $\langle \mathrm{F} \rangle p$ is true. At the metalinguistic level we deny the law of bivalence, saying that although $\langle \mathrm{F} \rangle p \vee \neg\langle \mathrm{F} \rangle p$ is valid neither $\langle \mathrm{F} \rangle p$ is neither true nor false at m_0 in $\mathcal{M}_1$. But at the object level, Semantic Rule 3.1 makes $[\mathrm{T}]\langle \mathrm{F} \rangle p \vee [\mathrm{T}]\neg\langle \mathrm{F} \rangle p$ valid.[8]

In 1970, only two interpretations of truth had occurred to me: Semantic Rule 3.1 and the following alternative, which reflects the supervaluational policy.

Rule 3.2.

$m, h \models [\mathrm{T}]\phi$ iff $m, h' \models \phi$ for all h through m

The definitions of satisfaction, implication and validity for this second semantic rule for truth go as follows.

Definition 3.2. Satisfaction$_2$, $\Vdash_2$, valid$_2$.

$\mathcal{M}, m, h \models_2 \phi$ is characterized for $\mathcal{L}_1$ by the Base Semantic Rules together with Semantic Rule 3.2. And as usual, $\mathcal{M}, m \models_2 \phi$ iff $\mathcal{M}, m, h \models_2 \phi$ for all $h \in \mathcal{H}_m$.

$\Gamma \Vdash_2 \phi$ iff for all frames $\mathcal{F} = \langle M, \prec \rangle$, $\mathcal{M}, m \models_2 \phi$ for all models $\mathcal{M}$ on $\mathcal{F}$ and all $m \in M$.

$\Vdash_2 \phi$ (ϕ is *valid*$_2$) iff $\emptyset \Vdash_2 \phi$.

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[8] I am indebted to Eli Hirsch for email correspondence in 2001 in which he brought this incongruity forcefully to my attention, as well as the problems discussed below in Section 4.

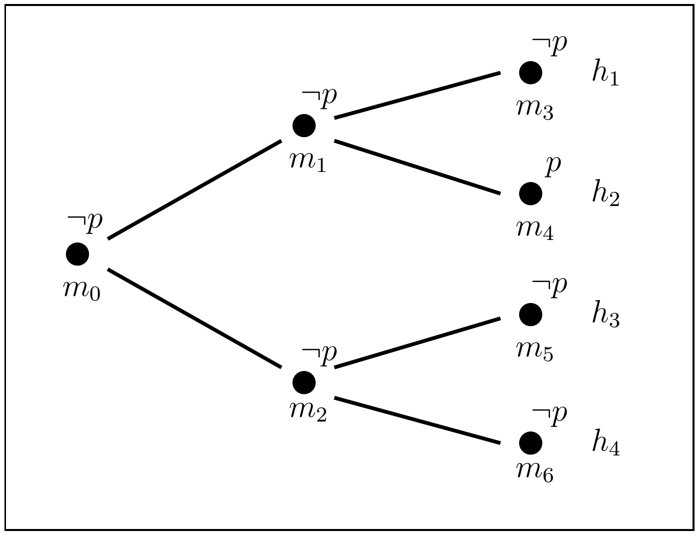
I chose Semantic Rule 3.1 because Semantic Rule 3.2 seemed problematic to me at the time I wrote the 1970 paper. I can find no written record of what these problems were and now, over thirty-seven years later, I can't speak with complete confidence about what was foremost in my mind. My best guess is that, thinking of other uses of supervaluations and in particular of van Fraassen's reconstruction of Strawsonian presupposition, I suspected that $p \leftrightarrow [\mathrm{T}]p$ in fact should be valid at the object level, even though at the metalinguistic level it is not. Also, I may have wanted to show that you could maintain an indeterminist position about truth in branching time, even with object level validities like $[\mathrm{T}]p \vee [\mathrm{T}]\neg p$—something that I still think is useful to know.

But considerations having to do with other applications of supervaluations to semantic problems, and especially considerations having to do with presupposition, have lost in the intervening years any force that they may have had in the late 1960s. And the incongruity that Semantic Rule 3.1 induces between metalinguistic preaching and object-level practice is certainly jarring. It would be far more interesting to know whether the supervaluational metalinguistic approach to truth in indeterminist models can be reconciled with object-level supervaluationism than to know that it is formally coherent to endorse Semantic Rule 3.1 in a supervaluational framework. These reasons speak for a reexamination of Semantic Rule 3.2. However, this interpretation of object-level truth raises problems that go beyond the use of supervaluations to interpret the base language. These problems involve new intuitions about the interaction between truth, historical necessity, and tenses in inteterministic settings, and will require us to rethink fundamental aspects of the semantic interpretation.

4. The problem of truth after the fact

Semantic Rule 3.1 makes $[\mathrm{T}]\phi$ semantically equivalent to ϕ, whereas Semantic Rule 3.2 makes $[\mathrm{T}]\phi$ semantically equivalent to $[\mathrm{H}]\phi$. That is, for all models $\mathcal{M}$, we have $\mathcal{M}, m, h \models_2 [\mathrm{T}]\phi$ iff $\mathcal{M}, m, h \models_2 [\mathrm{H}]\phi$, for all moments m and histories h of the model's frame.

Assume Semantic Rule 3.2 for the time being, and consider the following model, with particular attention to the behavior of $[\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p$ at m_4.



Model $\mathcal{M}_2$

Now, $\phi \to [\mathrm{P}]\langle\mathrm{F}\rangle\phi$ is valid in linear tense logic, so it is valid in indeterminist $\mathcal{L}_0$. But the intuitions that support this validity apply also to truth; that is, you should feel that $\phi \to [\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle\phi$ is valid. Once p has happened, it was not only always going to happen, but it was always true that it was going to happen.

For instance, consider m_4 in Model $\mathcal{M}_2$; p is true here, so it seems that $[\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p$ should be true at m_4. However, $\mathcal{M}_2, m_4 \not\models_2 [\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p$, because, for instance, $\mathcal{M}_2, m_0 \not\models_2 [\mathrm{T}]\langle\mathrm{F}\rangle p$. (This is because, for instance, $\mathcal{M}_2, m_0, h_1 \not\models_2 \langle\mathrm{F}\rangle p$.)

We have a similar problem from the perspective of m_0. Intuitively, the conditional $\langle\mathrm{F}\rangle p \to [\mathrm{F}]\langle\mathrm{P}\rangle[\mathrm{T}]\langle\mathrm{F}\rangle p$ should be true at m_0. If p will occur, then it will always have been true that p will occur. However, $\mathcal{M}_2, m_0 \not\models_2 \langle\mathrm{F}\rangle p \to [\mathrm{F}]\langle\mathrm{P}\rangle[\mathrm{T}]\langle\mathrm{F}\rangle p$.

With the interpretation of indeterminist tense logic that goes along with Semantic Rule 3.2, we have matched the metalinguistic behavior of truth in the object language, but have clashed with intuitions about truth after the fact.

5. A double-indexing solution

Consider again the problem of evaluating $[\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p$ at m_4 in $\mathcal{M}_2$. The calculation of satisfaction takes us from m_4 to m_1 and m_0. At these earlier moments, we evaluate $[\mathrm{T}]\langle\mathrm{F}\rangle p$, which forces us to look at histories passing through m_1 and m_0 other than h_2. It is this that makes $[\mathrm{T}]\langle\mathrm{F}\rangle p$ false at m_0 and m_1. However, this process of evaluation begins at m_4. If we were able to retain m_4 as a posited future moment as we move an evaluation into its past, we might be able to formalize the idea that the future up to m_4 is somehow settled from the standpoint of an earlier moment that is reached in this way from m_4 as part of an evaluation process.

We can do this by double-indexing. Instead of the satisfaction relation $m, h \models \phi$, which evaluates a formula relative to a moment and a history passing through that moment, we will use a relation $m_1, m_2, h \models \phi$, which evaluates formulas relative to two moments m_1 and m_2, and a history h passing through both. Here, m_1 is the *occurrence moment*—the moment at which the evaluated formula is assessed for truth. And m_2 is the *postulated future moment*—a moment either identical to m_1 or subsequent to m_1, representing an outcome which is supposed to be settled at this point in the evaluation.

The idea, then, will be to begin evaluating $[P][T]\langle F\rangle p$ at $\langle m_4, m_4, h_2\rangle$. At the beginning of the evaluation, the occurrence moment is the same as the postulated future moment. Then, using the semantic rule for $[P]$, $[T]\langle F\rangle p$ will be evaluated at $\langle m_1, m_4, h_2\rangle$. This leads to an evaluation of $\langle F\rangle p$ at $\langle m_1, m_4, h\rangle$ for all histories passing through m_4. Since there is only one such h, and since $m_4 \in V(p)$, $[T]\langle F\rangle p$ is satisfied at $\langle m_1, m_4, h_2\rangle$. For the same reason, $[T]\langle F\rangle p$ is satisfied at $\langle m_0, m_4, h_2\rangle$. Therefore, $[P][T]\langle F\rangle p$ is satisfied at $\langle m_4, m_4, h_2\rangle$.

To formalize this idea, we will need to define a double-indexed satisfaction relation $\models_3$, which tracks the postulated future moment as well as the occurrence moment: '$\mathcal{M}, m_1, m_2, h \models_3 \phi$' will mean that ϕ is satisfied in model $\mathcal{M}$ at $\langle m_1, m_2, h\rangle$, where m_1 is the occurrence moment and m_2 is the postulated future moment. Truth *simpliciter* is defined using supervaluations, exactly as before: $\mathcal{M}, m \models_3 \phi$ iff $\mathcal{M}, m, h \models_3 \phi$ for all $h \in \mathcal{H}_m$. But now, $\mathcal{M}, m, h \models_3 \phi$ iff $\mathcal{M}, m, m, h \models_3 \phi$—a formula is satisfied at a single moment and a history if and only if it is satisfied at this moment and history, when the moment is identified with *both* the occurrence moment and the postulated future moment.

Except for the addition of supervaluations, this interpretation strategy is exactly like the one that David Kaplan articulated in [Kaplan, 1978] to deal with satisfaction in formalized languages equipped with indexicals ***I*** and ***here***, as well as epistemic modalities $[M]$ and $\langle M\rangle$. ($[M]p$ is read "p must be the case", $\langle M\rangle p$ is read "p might be the case".)[9]

Kaplan's problem was to validate 'I am here' while allowing 'I might not be here' to be satisfiable. He solved this problem by evaluating formulas using a *postulated possible world* and an *actual world* (the terms are mine—I use them only to clarify the relation to my temporal indices). The formula ***here***(***I***) is satisfied at $\langle w_1, w_2\rangle$ iff the speaker in w_2 (the actual world) is located in w_1 (the postulated possibility) at the speaker's location in w_2. A formula is satisfied *simpliciter* at w iff it is satisfied at $\langle w, w\rangle$, and is valid if it is satisfied at every world in every model. Then ***here***(***I***) is valid, since it will always be satisfied at $\langle w_1, w_2\rangle$ when $w_1 = w_2$. But $[M]$***here***(***I***) is invalid, since ***here***(***I***) can be false at $\langle w_1, w_2\rangle$ when $w_1 \neq w_2$.

This approach to truth and satisfaction makes use of the idea that, although satisfaction of *entire formulas* depends on just one parameter (a world), the compositional semantic rules for satisfaction need to depend on two parameters (two worlds). It is this device that allows us to record the initial conditions that determine the references of ***I*** and ***here*** as we move to other worlds in evaluating modal operators. Similarly, I want to say that, although an entire formula such as $[P][T]\langle F\rangle p$ is satisfied *simpliciter* at a single moment, we need to invoke satisfaction relative to two moments in order to work out its satisfaction conditions. This temporal use of double-indexing is somewhat more complicated than Kaplan's indexical case, since the postulated future moment is not always held constant throughout the course of an evaluation. For example, in evaluating $\langle F\rangle\langle P\rangle[T]\langle F\rangle\langle F\rangle p$ at m_0 and h_2 in Model $\mathcal{M}_2$, we begin with $\langle m_0, m_0, h_2\rangle$, and pass in succession through $\langle m_4, m_4, h_2\rangle$, $\langle m_0, m_4, h_2\rangle$, $\langle m_0, m_4, h_2\rangle$, $\langle m_1, m_4, h_2\rangle$, and $\langle m_4, m_4, h_2\rangle$.

[9]Kaplan also incorporated (linear) tenses and ***now*** in his language, but to simplify things I leave the temporal dimension out.

These ideas are formalized in a definition of satisfaction for the temporal language $\mathcal{L}_2$, along the lines of the definitions in Section 3, but with an extra parameter for the postulated future moment. Frames, histories, valuations, and models are unchanged, remaining as defined in 2.1. Definition 3.1 gives way to the following double-indexed satisfaction definition.

Definition 5.1. $\mathcal{M}, m, h \models_3 \phi$.

1. **Basis:** If $\phi \in \mathcal{P}$ then $\mathcal{M}, m_1, m_2, h \models_3 \phi$ iff $m_1 \in V(\phi)$.
2. **Booleans:** Boolean conditions are routine.
3. **Past:** $\mathcal{M}, m_1, m_2, h \models_3 \langle P \rangle \phi$ iff for some $m' \prec m_1$, $\mathcal{M}, m', m_2, h \models_3 \phi$.
   $\mathcal{M}, m_1, m_2, h \models_3 [P]\phi$ iff for all $m' \prec m_1$, $\mathcal{M}, m', m_2, h \models_3 \phi$.
4. **Future:** $\mathcal{M}, m_1, m_2, h \models_3 \langle F \rangle \phi$ iff for some m', $m_1 \prec m'$ and
   $m' \in h$, $\mathcal{M}, m', m_2', h \models_3 \phi$, where $m_2' = \max(m_2, m')$.
   $\mathcal{M}, m_1, m_2, h \models_3 [F]\phi$ iff for all m', $m' \prec m$ and $m' \in h$,
   $\mathcal{M}, m', m_2', h \models_3 \phi$, where $m_2' = \max(m_2, m')$.
5. **Historical Necessity:** $\mathcal{M}, m_1, m_2, h \models_3 [F]\phi$ iff for all h' passing through m_1,
   $\mathcal{M}, m_1, m_2, h' \models_3 \phi$.
6. **Truth:** $\mathcal{M}, m_1, m_2, h \models_3 [T]\phi$ iff for all h' passing through m_2,
   $\mathcal{M}, m_1, m_2, h' \models_3 \phi$.

Semantic Rules for Double-Indexed Indeterminist Tense Logic

Satisfaction$_3$ *simpliciter* is defined in two stages: Kaplan first, then van Fraassen.

Definition 5.2. $\mathcal{M}, m \models_3 \phi$, $\Vdash_2$, valid$_3$.

$\mathcal{M}, m, h \models_3 \phi$ iff $\mathcal{M}, m, m, h \models_3 \phi$

$\mathcal{M}, m \models_3 \phi$ iff $\mathcal{M}, m, h \models_3 \phi$ for all $h \in \mathcal{H}_m$.

$\Gamma \Vdash_2 \phi$ iff for all frames $\mathcal{F} = \langle M, \prec \rangle$, $\mathcal{M}, m \models_3 \phi$ for all models $\mathcal{M}$ on $\mathcal{F}$ and all $m \in M$.

$\Vdash_2 \phi$ (ϕ is *valid*$_3$) iff $\emptyset \Vdash_2 \phi$.

6. Some formal properties of the double-indexed system

Our intention is that whenever $\langle m_1, m_2, h \rangle$ is visited in the course of evaluating a formula, $m_1, m_2 \in h$ and $m_1 \preceq m_2$. That intention is realized in this logic.

Definition 6.1. *Triples*(ϕ, m, h).

Let $\mathcal{M}$ be a model on $\mathcal{F} = \langle M, \preceq \rangle$, let h be a history on $\mathcal{F}$, let $m \in h$, and let ϕ be a formula of $\mathcal{L}_2$. We define the set of triples relevant to the evaluation of ϕ at m and h by the following induction on the complexity of ϕ.

Basis. If $p \in \mathcal{P}$, then $\mathit{Triples}(p, m, h) = \{\langle m, m, h \rangle\}$.

Booleans. $\mathit{Triples}(\neg\phi) = \mathit{Triples}(\phi)$, and
$\mathit{Triples}(\phi \rightarrow \psi) = \mathit{Triples}(\phi) \cup \mathit{Triples}(\psi)$. Similarly for other boolean connectives.

Tense operators.
$\mathit{Triples}(\langle \mathrm{F} \rangle \phi) = \mathit{Triples}([\mathrm{F}]\phi) = \mathit{Triples}(\phi) \cup \{\langle m_1, m_2, h \rangle$ / for some $\langle m_1', m_2', h' \rangle \in \mathit{Triples}(\phi)$, $m_1' \prec m_1$, $m_2 = \max(m_1, m_2')$, and $h = h'\}$.
$\mathit{Triples}(\langle \mathrm{P} \rangle \phi) = \mathit{Triples}([\mathrm{P}]\phi) = \mathit{Triples}(\phi) \cup \{\langle m_1, m_2, h \rangle$ / for some $\langle m_1', m_2', h' \rangle \in \mathit{Triples}(\phi)$, $m_1 \prec m_1'$, $m_2 = m_2'$, and $h = h'\}$.

Modal operators.
$\mathit{Triples}([\mathrm{H}]\phi) = \mathit{Triples}(\langle \mathrm{H} \rangle \phi)\, \mathit{Triples}(\phi) \cup \{\langle m_1, m_2, h \rangle$ / for some $\langle m_1', m_2', h' \rangle \in \mathit{Triples}(\phi)$, $m_1 = m_1'$, $m_2 = m_2'$, and $h' \in \mathcal{H}(m_1')\}$.
$\mathit{Triples}([\mathrm{T}]\phi) = \mathit{Triples}(\phi) \cup \{\langle m_1, m_2, h \rangle$ / for some $\langle m_1', m_2', h' \rangle \in \mathit{Triples}(\phi)$, $m_1 = m_1'$, $m_2 = m_2'$, and $h' \in \mathcal{H}(m_2')\}$.

The following remark, which shows that every triple visited in the evaluation of a formula meets the requirement we mentioned above, is easily proved by induction on ϕ.

Remark 6.1. If $\langle m_1, m_2, h \rangle \in \mathit{Triples}(\phi)$, then $m_1 \preceq m_2$ and $m_1, m_2 \in h$.

Furthermore, double-indexed and basic implication agree on the [T]-free fragment. The proof is straightforward and is omitted.

Remark 6.2. Let Γ be a set of formulas of $\mathcal{L}_1$ and ϕ be a formula of $\mathcal{L}_1$. Then $\Gamma \Vdash_3 \phi$ iff $\Gamma \Vdash_2 \phi$ iff $\Gamma \Vdash_1 \phi$ iff $\Gamma \Vdash \phi$.

However, when truth is added to the object language, the logics become incomparable: neither the set of valid$_2$ formulas nor the set of valid$_3$ formulas is contained in the other.

Remark 6.3. $\Vdash_3 p \rightarrow [\mathrm{P}][\mathrm{T}]\langle \mathrm{F} \rangle p$, but $\nVdash_2 p \rightarrow [\mathrm{P}][\mathrm{T}]\langle \mathrm{F} \rangle p$.

Remark 6.4. $\Vdash_2 \langle \mathrm{P} \rangle [\mathrm{T}] \langle \mathrm{F} \rangle p \rightarrow \langle \mathrm{P} \rangle [\mathrm{H}] \langle \mathrm{F} \rangle p$, but $\nVdash_3 \langle \mathrm{P} \rangle [\mathrm{T}] \langle \mathrm{F} \rangle p \rightarrow \langle \mathrm{P} \rangle [\mathrm{H}] \langle \mathrm{F} \rangle p$.

Note also, that although $\nVdash_3 \langle \mathrm{P} \rangle [\mathrm{T}] \langle \mathrm{F} \rangle p \rightarrow \langle \mathrm{P} \rangle [\mathrm{H}] \langle \mathrm{F} \rangle p$, we do have $\Vdash_3 [\mathrm{T}]\langle \mathrm{F} \rangle p \rightarrow [\mathrm{H}]\langle \mathrm{F} \rangle p$. In fact, $[\mathrm{T}]\langle \mathrm{F} \rangle \phi$ and $[\mathrm{H}]\langle \mathrm{F} \rangle \phi$ are equivalent in the double-indexed system.

Remark 6.5. $\Vdash_3 [\mathrm{T}]\langle \mathrm{F} \rangle \phi \leftrightarrow [\mathrm{H}]\langle \mathrm{F} \rangle \phi$.

The formula $\langle \mathrm{F} \rangle p \rightarrow [\mathrm{T}]\langle \mathrm{F} \rangle p$ is of some interest. It is invalid$_3$, although it can never be false.

Remark 6.6. $\mathcal{M}_2, m_0 \not\models_3 \langle \mathrm{F} \rangle p \rightarrow [\mathrm{T}]\langle \mathrm{F} \rangle p$. However, $\mathcal{M}, m \not\models_3 \neg(\langle \mathrm{F} \rangle p \rightarrow [\mathrm{T}]\langle \mathrm{F} \rangle p)$, for any model $\mathcal{M}$ and moment m of $\mathcal{M}$.

7. Comparison with MacFarlane's Solution

In [MacFarlane, 2002], and in an as-yet unpublished sequel to this paper [MacFarlane, 2006], John MacFarlane discusses issues similar to those with which I am concerned in this paper, and proposes a solution that draws on similar ideas but that packages them differently. The packaging difference turns out to be substantive: it affects the notion of validity that the formal theory delivers, and its philosophical interpretation.

MacFarlane confines his explicit semantics to a tense logic with (apparently) only the operators $\langle F \rangle$ and $[H]$.[10]

For purposes of comparing MacFarlane's theory with mine, I will begin with the language $\mathcal{L}_1$; that is, the language will include tense operators and $[H]$, but will omit $[T]$ for the time being. MacFarlane never gives explicit semantic rules for the entire language, but I think it is clear what these rules would be. MacFarlane is working with a satisfaction relation $\models_3$ that depends on a model, a history, and two moments, which he calls "the context of evaluation" and "the context of assessment." MacFarlane's context of evaluation corresponds to my occurrence moment and his context of assessment corresponds to my postulated future moment.[11]

The recursive definition of $\mathcal{M}, m_1, m_2, h \models_3 \phi$ (where m_1 is the e-context m_2 the a-context) incorporates the Base Semantic Rules given above for $\models$, with 'm_2' playing an inert role in the recursion; this parameter is held constant in the evaluation of formulas. For instance, the rules for future tense and historical necessity would be:

4′. **Future:** $\mathcal{M}, m_1, m_2, h \models_4 \langle F \rangle \phi$ iff for some m', $m_1 \prec m'_1$ and $m'_1 \in h$, $\mathcal{M}, m'_1, m_2, h \models_4 \phi$.
$\mathcal{M}, m_1, m_2, h \models_4 [F]\phi$ iff for all m'_1, $m'_1 \prec m_1$ and $m'_1 \in h$, $\mathcal{M}, m'_1, m_2, h \models_4 \phi$.

5′. **Historical Necessity:** $\mathcal{M}, m_1, m_2, h \models_4 [F]\phi$ iff for all h' passing through m_1, $\mathcal{M}, m_1, m_1, h' \models_4 \phi$.

Future Tense and Historical Necessity Rules for $\models_4$

Implication and validity are then defined as follows.

Definition 7.1. $\mathcal{M}, m_1, m_2 \models_4 \phi$, $\Gamma \Vdash_4 \phi$, valid$_4$.

$\mathcal{M}, m_1, m_2 \models_4 \phi$ iff $\mathcal{M}, m, m, h \models_4 \phi$ for all h passing through m_2.

$\Gamma \Vdash_4 \phi$ iff for all frames $\mathcal{F} = \langle M, \prec \rangle$, $\mathcal{M}, m_1, m_2 \models_4 \phi$ for all models $\mathcal{M}$ on $\mathcal{F}$ and all $m_1, m_2 \in M$ such that $m_1 \preceq m_2$.

$\Vdash_4 \phi$ (ϕ is *valid*$_4$) iff $\emptyset \Vdash_4 \phi$.

Once more, consider Model $\mathcal{M}_2$. We imagine an evaluator occupying, say, a position at m_4 (occupying it either in fact, or simply postulating it as a perspective). From this

[10] [MacFarlane, 2002] uses a modal syntax like the one I describe in this paper, while [MacFarlane, 2006] treats modal operators as quantifiers. For purposes of comparison, I will confine myself to modal object languages. The difference between modal and quantificational tense languages is not important for the purposes of this paper.

[11] I have been speaking all along of the process of evaluating a formula, using 'evaluate' in quite a different sense. To avoid confusion, from here on I will use the terms 'a-context' and 'e-context' for MacFarlane's two context.

standpoint, the evaluator asks whether a formula ϕ obtains at a previous moment, say m_0. Then we are in effect asking whether $\mathcal{M}, m_1, m_2, h_2 \models_4 \phi$. For instance, MacFarlane's satisfaction definition provides a way of explaining why, from the perspective of m_4 in this model, $\langle P \rangle \langle F \rangle p$ is true. In fact, we have $\mathcal{M}_2, m_4, m_4, h_2 \models_4 \langle P \rangle \langle F \rangle p$ (even more, we have $\mathcal{M}_2, m_4, m_4, h_2 \models_4 [H] \langle P \rangle \langle F \rangle p$). This is because for all histories h passing through m_4, $\mathcal{M}_2, m_0, m_4, h \models_4 \langle F \rangle p$.

Using this account of satisfaction, we can explain the apparent validity of $p \rightarrow [P][T]\langle F \rangle p$ as follows.

Remark 7.1. Let $\mathcal{M}$ be a model and m_1 and m_2 be moments of $\mathcal{M}$ with $m_1 \preceq m_2$. Let $\mathcal{M}, m_1, m_2 \models_4 p$. Then, for any $m \prec m_1$, $\mathcal{M}, m, m_2 \models_4 \langle F \rangle p$.

In fact, let $\mathcal{M}, m_1, m_2 \models_4 p$ and $m \prec m_1$, and let h be any history passing through m_2. Then $\mathcal{M}, m, m_2, h \models_4 \langle F \rangle p$.

The validity that is established in Remark 7.1 uses metalinguistic truth, as well as a metalinguistic form of past tense. This formulation is essential.

Without a way of expressing truth in the object language, MacFarlane's account can provide explanations of validities involving truth and tense only by using the metalinguistic conception of truth that is incorporated in the definition of $\models_4$. For instance, the explanation of why 'It was true that p was going to happen' sounds intuitively correct from the standpoint of m_4 in Model $\mathcal{M}_2$ is simply that $\mathcal{M}_2, m_0, m_4, h_2 \models_4 \langle F \rangle p$.

This approach suffers from two connected defects: (1) it is contrary to the spirit of tense logic, since it explains the past 'was' in 'It was true' by appropriate choices of contextual moments, rather than by semantic rules for tense operators, (2) it cannot account for the intuitive validity of $p \rightarrow [F][T]\langle F \rangle p$, $\langle F \rangle p \rightarrow \langle F \rangle [P][T]\langle F \rangle p$, and $[F](p \rightarrow [P][T]\langle F \rangle p)$. Metalinguistic truth cannot be embedded in object-level tense operators; when truth is confined to the metalanguage, we can't develop an explicit theory of its interactions with tense operators.

We obtain a better comparison with our double-indexed implication $\Vdash_4$ if we add truth to the object language of MacFarlane's theory—that is, if we compare his theory to ours on the language $\mathcal{L}_2$. Of course, MacFarlane does not state a semantic rule for [T], but the following rule seems to be the only possibility.

6′. **Truth:** $\mathcal{M}, m_1, m_2, h \models_4 [T]\phi$ iff for all h' passing through $\max(m_1, m_2)$, $\mathcal{M}, m_1, m, h' \models_4 \phi$, where $m = \max(m_1, m)$.

Truth Rule for $\models_4$

To understand this rule, it is important to note that although every evaluation of a formula in MacFarlane's system begins at a point $\langle m_1, m_2, h \rangle$, with $m_1 \preceq m_2$ and $m_1, m_2 \in h$, later stages of evaluation may very well visit points $\langle m'_1, m'_2, h' \rangle$, where $m_2 \prec m_1$. This happens, for instance, in the evaluation of $\langle F \rangle p$ at $\langle m_0, m_0, h_2 \rangle$ in Model $\mathcal{M}_2$, where we will need to evaluate $\mathcal{M}_2, m_4, m_0, h_4 \models_4 p$. MacFarlane intends the a-context to serve as a retrospective perspective from which earlier utterances are assessed, so it is not entirely clear what to do in this case, although it does seem clear that we do not want both to hold the a-context fixed throughout an evaluation and to use the a-context to interpret [T]. I have chosen to take the maximum of the two moments rather than to allow MacFarlane's context of evaluation to change in the course of an evaluation.

One of the intuitive validities that we used to motivate the double-indexed system, $p \rightarrow [\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p$ is in fact valid in the extended version of MacFarlane's system.

Remark 7.2. $\Vdash_4 p \rightarrow [\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p$.

In fact, let $\mathcal{M}, m_1, m_2, h \models_4 p$, where $m_1 \preceq m_2$ and $m_2 \in h$, and let $m_1' \prec m_1$. Then $\mathcal{M}, m_1', m_2, h' \models_4 \langle\mathrm{F}\rangle p$ for all h' such that $m_2 \in h'$. Therefore, $\mathcal{M}, m_1', m_2, h \models_4 [\mathrm{T}]\langle\mathrm{F}\rangle p$.

However, MacFarlane's system does not validate many other formulas that the double-indexed system validates—for instance, $\langle\mathrm{F}\rangle p \rightarrow \langle\mathrm{F}\rangle[\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p$ and $[\mathrm{F}][\mathrm{P}](p \rightarrow [\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p)$.

Remark 7.3. $\mathcal{M}_2, m_0, m_0, h_2 \not\models_4 \langle\mathrm{F}\rangle p \rightarrow \langle\mathrm{F}\rangle[\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p$

In fact, $\mathcal{M}_2, m_0, m_0, h_2 \models_4 \langle\mathrm{F}\rangle p$ because $\mathcal{M}_2, m_4, m_0, h_2 \models_4 p$. But $\mathcal{M}_2, m_0, m_0, h_2 \not\models_4 \langle\mathrm{F}\rangle[\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p$ because (1) if $m = m_1$, $\mathcal{M}_2, m, m_0, h_2 \not\models_4 [\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p$ because $\mathcal{M}_2, m_0, m_0, h_1 \not\models_4 \langle\mathrm{F}\rangle p$ and (2) if $m = m_4$, $\mathcal{M}_2, m, m_0, h_2 \not\models_4 [\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p$ for the same reason.

Remark 7.4. $\mathcal{M}_2, m_0, m_0, h_2 \not\models_4 [\mathrm{F}][\mathrm{P}](\langle\mathrm{F}\rangle p \rightarrow [\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p)$

In fact, $\mathcal{M}_2, m_1, m_0, h_2 \models_4 \langle\mathrm{F}\rangle p$. And $\mathcal{M}_2, m_1, m_0, h_2 \not\models_4 [\mathrm{P}][\mathrm{T}]\langle\mathrm{F}\rangle p$, because $\mathcal{M}_2, m_0, m_0, h_2 \not\models_4 [\mathrm{T}]\langle\mathrm{F}\rangle p$, because $\mathcal{M}_2, m_0, m_0, h_1 \not\models_4 \langle\mathrm{F}\rangle p$.

8. Philosophical ramifications

Although I certainly believe that logic can be indispensable in clarifying philosophical thinking, I have thought for a long time that more or less straightforward connections between logical results and substantive philosophical consequences are rare. The connection that John MacFarlane draws in [MacFarlane, 2006] between the requirements of an adequate semantic theory of indeterminist tense logic and the need to relativize truth to a "context of assessment" as well as a "context of evaluation" illustrates the point. There is a kind of complementarity here between the requirements of a formally adequate logical theory of tenses and truth in an indeterminist setting, and the force of the philosophical conclusions that you can draw from the logic.

Throughout the 2002 paper, MacFarlane refers to intuitions about the truth of utterances and assertions in various contexts. At some points, it actually seems as if utterances are awarded an explicit place in the semantic theory. I want to set MacFarlane's remarks about utterances aside at the outset, because I do not think that they are helpful. I do believe that it may be useful to bring utterances into consideration in pursuing some pragmatic purposes. But in semantic inquiries like this one, which is concerned with truth and tense in indeterminist settings, I think it is an irrelevant distraction.

All along, MacFarlane works with expressions that have a compositional semantics. Assuming the availability of such expressions is perfectly appropriate in a semantic project. But, if utterances are construed as actual or possible events, they do not deliver disambiguated, structured sentences. Utterances can be ambiguous, incompletely intelligible, truncated, and ill-formed. In associating a disambiguated, syntactically structured sentence with an utterance one is making a considerable abstraction from the event itself, especially if the pragmatic interpretation of the event is left implicit.

Utterance events are not always available or even possible in many cases where we want to think about truth. It is legitimate to imagine a world in which sentient life evolved and a time in this world before this evolution has occurred, and to judge the truth of sentences at that moment in this world.

And even if we start out by observing or imagining an utterance event, along with as much detail as you like about the accompanying circumstances, the event and the circumstantial detail is left behind when semantic intuitions are brought to bear on examples. As long as our example has what is required to reconstruct a context and a disambiguated sentence, our judgments about the truth of the utterance, as far as I can see, will be the same as our judgments about the truth of the sentence in the context. Since we can make the latter sort of judgments without bringing in utterances in the first place, utterances and assertions are pretty much irrelevant for semantic purposes.

Of course, the word 'utterance' exhibits the process-product ambiguity. It can mean the event of uttering, or what is uttered. With the latter meaning, and what what is uttered understood as a syntactically structured, disambiguated sentence, we can think of semantics as a theory of utterances in context. Although I doubt that he meant to be construed in this way, for most purposes I think it is possible to understand MacFarlane to be drawing no distinction between utterances and sentences in context.

MacFarlane's chief conclusion in the 2002 paper is that to do justice to semantic intuitions about future contingent statements, we must give up what he calls the principle of "the absoluteness of utterance-truth." ([MacFarlane, 2006][p. 322].). This principle claims that the truth of an utterance should depend only on the context of utterance. His reasons for abandoning this principle are considerations, based mainly on variations of Aristotle's Sea-Battle example, of the sort I used in Section 4, above, to motivate a double-indexed semantic theory of indeterminist tense and truth.

I have no wish to defend the absoluteness principle. As I said, I believe that the issues with which MacFarlane and I are concerned are semantic, and utterances are irrelevant. Moreover, semantic theory is not really a branch of philosophy—it belongs to the intersection of linguistics and logic, and is governed by the need to put together a formally adequate theory that is as elegant as possible and that does justice to the intuitions and evidence. Generalizations about how to proceed may emerge from studying the best work in the field, but principles—especially philosophical principles—are secondary at best.

But the double-indexed approach developed in this paper does seem to undermine any reasons for following MacFarlane in postulating a separate "context of evaluation" for the semantic evaluation of sentences in a context or for the definition of logical consequence.

In indeterminist tense logic, a context is a moment-history pair $\langle m, h \rangle$. The point that now needs to be made is delicate. In the recursive evaluation of a sentence relative to a context, we need to vary m, but to keep track of the history of the values that m has assumed in the course of evaluation. The evaluation of past tense, for instance, forces us to consider other values of m that are earlier in time, but in evaluating an embedded truth operator we do not want to forget the current value. Since we do not in fact have to remember the entire evaluation history, double-indexing provides an elegant solution to this problem: we split the moment parameter of satisfaction into two parameters, writing '$\mathcal{M}, m_1, m_2, h \models_3 \phi$', and let the first position vary in interpreting tense, while the second position serves to remember an appropriate value.[12]

Now we can state the delicate point. There are two *argument positions* for moments available at all stages of the semantic evaluation of a sentence, including the initial one. But, since the values of these parameters are identified in the initial stage (see the first clause of Definition 5.2), only one moment is involved at this stage.

Let's interpret Model $\mathcal{M}_2$ to conform to the Sea Battle example. Then p stands for 'A sea battle occur'. (I use tenseless 'occur' here as a base form that combines with tenses to produce forms like 'A sea battle will occur', corresponding to $\langle F \rangle p$, and 'A sea battle occurred', corresponding to $\langle P \rangle p$.) MacFarlane's argument for needing a context of evaluation is then that, from the standpoint of m_1, it was true that a sea battle would occur, whereas from the standpoint of m_0, it is neither true nor false that a sea battle will occur. However, in presenting his argument, he is careful to shift to the truth of utterances, saying things like "The assertion that there would be a sea battle was true." Although, as I just showed, the point can be made simply and directly using indirect discourse 'true', MacFarlane always shifts to utterance or assertion truth.

Here, I think, reliance on utterances as if they were part of the theory begins to bite. Since we can state the problematic cases equally well (or even better) by saying things like 'It was true that a sea battle would occur', a full solution to this problem has to deal with locutions of this type, i.e. with interactions between indirect discourse truth, the tenses, and historical necessity. Since we are centrally concerned with inferences involving tenses, historical necessity, and indirect discourse truth, we need to have all these things in our object language. Once we do that, MacFarlane's semantic proposal—to introduce a second temporal index in the definition of supervaluational truth, but to continue to use a single index in the recursive semantic evaluation of sentences—is formally inadequate. As I showed above, this approach gives different validities than my double-indexed approach, and these validities do not conform to the intuitions that MacFarlane cites.

At the end of [MacFarlane, 2006], he mentions applications of the idea of a context of assessment to other phenomena of philosophical interest. If what I have said here is right, these applications will need independent motivation—motivation that MacFarlane may well be able to supply.

9. Other after-the-fact sensitive constructions

In this paper, I have concentrated my attention on what might be called "future-laden" constructions: sentences that make commitments about the future. I have developed a semantic model according to which we can say that these sentences may be neither true nor false at a moment, even though at a later moment we can also say that they *were true*.

I think that such constructions are more common than most philosophers might think. In [Thomason, 2007], I argue that 'know' is a source of future-laden sentences. Other such locutions, I believe, include conditionals, explicit causal constructions, and present progressive sentences. I hope to discuss these interesting cases in another work.

[12] There is a lucid explanation of the idea, using different terminology, in [Lewis, 1981].

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