Semi-empirical model for radar backscatter from snow at 35 and 94 GHz

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Abstract — Radar backscatter experiments were conducted at 35 and 94 GHz to measure the response of snow-covered ground to snow depth, liquid water content and ice crystal size. The measurements included observations over a wide angular range extending between normal incidence and 60° for all linear polarization combinations. A numerical radiative transfer model was developed and adapted to fit the experimental observations. Next, the radiative transfer model was exercised for a wide range of conditions and the data that was generated was used to develop relatively simple semi-empirical expressions that relate the backscattering coefficient (for each linear polarization) to incidence angle, snow depth, crystal size, and liquid water content.

INTRODUCTION

Mathematical complexity is a common feature of most electromagnetic scattering models. Whereas the development of complicated models is often necessary in order to adequately characterize the scattering process and understand the physical mechanisms giving rise to the observed scattering behavior, their complexity tends to limit their use to a narrow segment of the remote sensing community. The objective of this paper is to present a utilitarian semi-empirical model for the radar backscattering coefficient of snow at 35 and 95 GHz. By utilitarian, we mean that the model expressions and associated graphs are easy to use without the need for sophisticated computers and elaborate integration routines. The model is semi-empirical in the sense that its expressions were generated by fitting a data base to general analytical forms characteristic of approximate solutions of the scattering problem. The data base was itself generated using a numerical radiative transfer solution that had been verified against experimental observations. The model is limited to the 35- and 95-GHz atmospheric window frequencies because they are of particular interest for a number of applications.

In 1991, the results of an extensive study on the millimeter-wave radar backscatter response of snowcover were reported in a series of two papers [1, 2]. The first paper introduced a radiative transfer model for a snow layer above a ground surface. The model was then solved numerically (so as to include multiple-scattering contributions) in order to illustrate the general behavior of the co-polarized backscattering coefficients, σ°_{vv} and σ°_{hh}, and the cross-polarized backscattering coefficient, σ°_{hv}, as a function of some of the pertinent parameters of the snow medium, such as depth and liquid water content. In the second paper, calculations based on the model were compared with experimental measurements that had been made by a calibrated truck-mounted polarimetric scatterometer with operating frequencies at 35, 94, and 140 GHz. The radar measurements had been augmented with extensive measurements of snow depth, liquid water content, crystal size distribution, and snow density, among others. Fig. 1 is a typical example illustrating how well the theoretical model predictions agree with the experimental observations. The calculated continuous curves and associated experimental data correspond to diurnal observations made between 8 am and midnight. The decrease in σ°_{hh} after 11 am and the ensuing variation is in response to the diurnal variation of m_v. *In-situ* measurements of m_v indicated that the snow layer was dry prior to 11 am and after 8 pm.

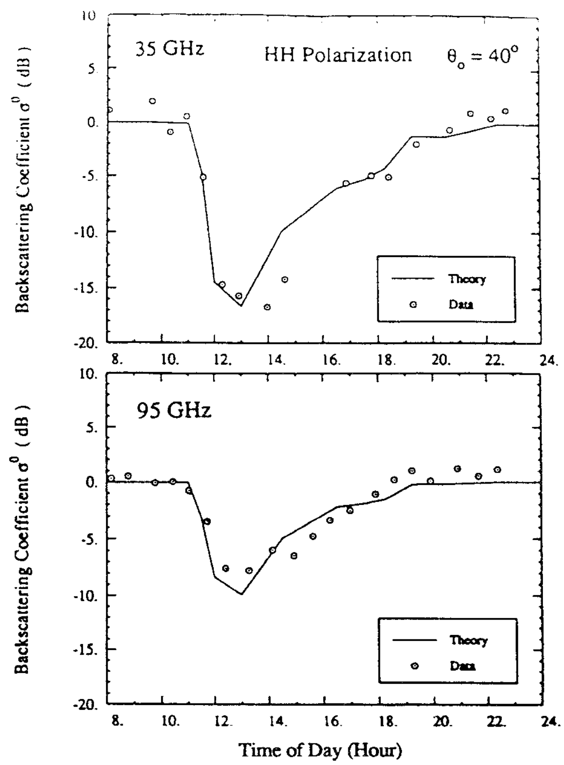

Figure 1: Comparison of measured and calculated diurnal response of σ^0 for snow at 35 and 95 GHz. The incidence angle is 40°, snow density is 0.32 g/cm^3, the snow depth is 12 cm, and mean crystal diameter is 1 mm [1, 2].

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SEMI-EMPIRICAL MODEL

Having demonstrated that the numerical solution of the radiative transfer model provides results that are in close agreement with experimental observations for a wide range of conditions, we then used the model to examine the general response of σ° to each of the critical radar and snow parameters. Based on this observed behavior, we constructed the following semi-empirical model:

$$\sigma^\circ_{ij} = A\left[1-\exp(-Bh\rho_s \sec\theta')\right] e^{-Cm_v^x}\cos\theta + \frac{D\Gamma_0 \exp(-\tan^2\theta/2m^2)}{2m^2\cos^4\theta}, \quad \mathrm{m^2/m^2} \quad (1)$$

where

i, j = v or h polarizations

h = snow depth in cm ($h \geq 10$ cm)

ρ_s = snow density in g/cm^3

m_v = snow volumetric liquid water content in %

m = surface rms slope

$$D = \begin{cases} 1 & \text{for hh and vv polarizations} \\ 0 & \text{for hv polarization} \end{cases}$$

The nadir reflectivity is given by

$$\Gamma_0 = \left|\frac{\sqrt{\epsilon_s}-1}{\sqrt{\epsilon_s}+1}\right|^2, \quad (2)$$

where ϵ_s is the permittivity of snow [3, p. 2071]:

$$\epsilon_s = 1 + 1.832\rho_s + 0.03m_v. \quad (3)$$

The angle θ' is the refraction angle in the snow medium and is related to θ by

$$\theta' = \sin^{-1}\left[\frac{\sin\theta}{\sqrt{\epsilon_s}}\right]. \quad (4)$$

The coefficients A, B, C, and x are empirically determined constants, of which A and B are functions of the mean ice crystal diameter d (in mm):

$$A = \begin{cases} A_0\left[1-\exp(-A_1 d^y)\right] & @\ 35\ \mathrm{GHz} \\ A_0 & @\ 94\ \mathrm{GHz} \end{cases} \quad (5)$$

$$B = \begin{cases} B_0\left[1-\exp(-B_1 d^z)\right](1+m_v) & @\ 35\ \mathrm{GHz} \\ B_0(1+m_v) & @\ 94\ \mathrm{GHz} \end{cases} \quad (6)$$

The values of A_0, A_1, B_0, B_1, C, x, y, and z are given in Table 1 for 35 and 94 GHz for each of the three linear polarization configurations. This model is applicable for the following ranges:

$10^\circ \leq \theta \leq 60^\circ$
$0 \leq m_v \leq 12\%$ (upper limit is 5% for hv polarization)
$0.2 \leq \rho_s \leq 0.5$ g/cm^3
$0.5 \leq d \leq 3$ mm
$0.1 \leq m \leq 0.8$
$h \geq 10$ cm

Table 1: Model coefficients.

	35 GHz			94 GHz		
	vv	hh	hv	vv	hh	hv
A_0	1.7	1.87	1	1.5	1.7	0.85
A_1	1.33	1.33	0.51	—	—	—
B_0	0.67	0.67	0.67	0.214	0.214	0.126
B_1	0.18	0.18	0.065	—	—	—
C	1.6	1.6	2.2	0.75	0.75	0.7
x	0.5	0.5	0.6	0.6	0.6	0.8
y	1.5	1.5	1.5	—	—	—
z	2.5	2.5	2.5	—	—	—

The errors between the values predicted by the semi-empirical model and either the numerical radiative transfer solution or experimental observations are typically on the order of 1–3 dB for $m_v \leq 5\%$. Larger errors may occur if $m_v > 5\%$, particularly for hv polarization. The model behavior is illustrated in Figs. 2 and 3 for vv and hv polarization for a snowpack containing ice crystals characterized by a Gaussian size distribution with a mean diameter d and a standard deviation of $0.2d$. The snow density is 0.3 g/cm^3 and the rms slope of the snow surface is 0.5.

CONCLUDING REMARKS

A semi-empirical model is presented for the radar backscattering coefficient of snow-covered ground at 35 and 94 GHz. For each linear polarization combination, the model expressions are given in terms of the incidence angle θ and four snow parameters: the depth h, density ρ_s, particle diameter d and wetness m_v. The model accuracy is on the order of 1–3 dB.

REFERENCES

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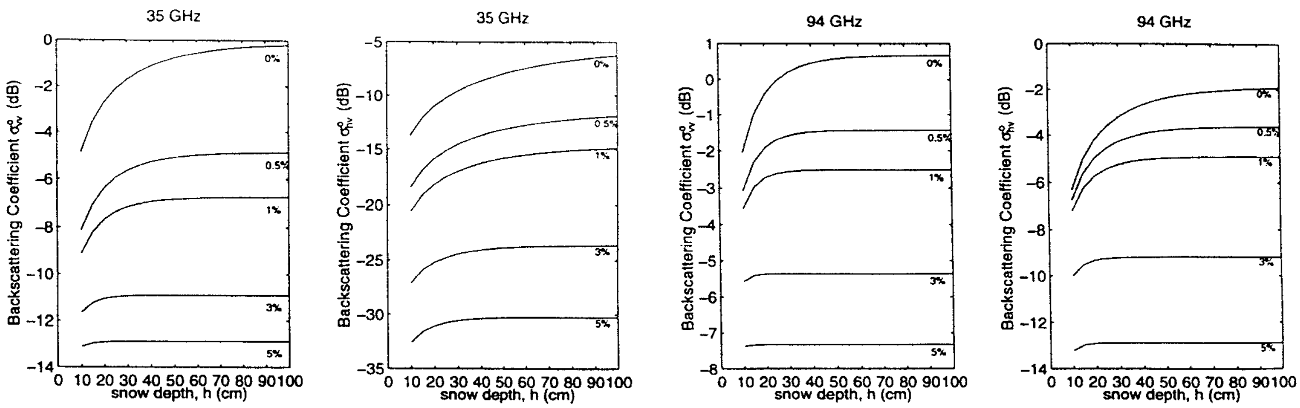

Figure 2: Backscatter response to snow depth for five different snow wetnesses. The incidence angle is 40° and the mean crystal diameter is 1 mm.

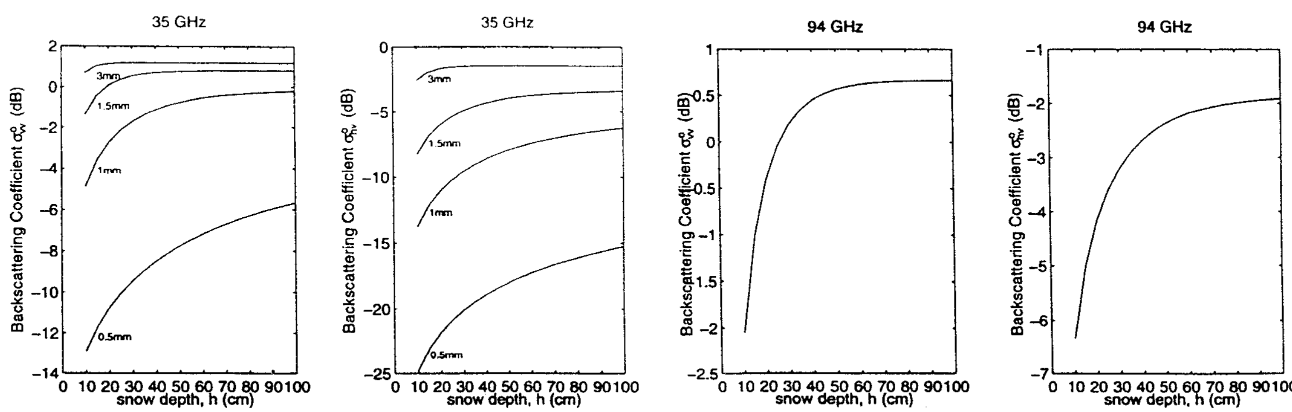

Figure 3: Backscatter response of dry snow to snow depth for four different particle diameters (because the dependence on particle diameter is weak at 94 GHz, only one set of curves is shown for $d = 1$ mm). Observation angle is 40°.