CALIBRATION OF A POLARIMETRIC SYNTHETIC APERTURE RADAR USING A KNOWN DISTRIBUTED TARGET

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Abstract- Existing methods for external calibration of polarimetric synthetic aperture radars (SAR) are all based on point calibration targets. The quantity of interest in radar measurement of distributed targets is the backscattering coefficient which is different from the radar cross section (RCS) formulated for point targets. Therefore, in order to infer the backscattering cross section of a distributed target from a point target rigorously, the polarimetric ambiguity function of the SAR is needed. In this paper a calibration algorithm is proposed that circumvents all problems associated with the point target calibration methods. It is shown that the radar distortion parameters and calibration constant can be obtained from a distributed target with known differential Mueller matrix. The distortion parameters are then used to provide the calibrated differential Mueller matrix for the other homogeneous targets in the image. This algorithm is tested for the JPL L- and C-band SAR using four different distributed targets measured with polarimetric scatterometers.

INTRODUCTION

Measurement of scattering matrix of a point target involves a comparison of the measured response of the unknown target with the measured response of one or more calibration targets of the known scattering matrices. In other words similar quantities (radar cross section area) are measured and compared. The situation is markedly different when distributed targets are being measured. The quantity of interest in this case is the backscattering coefficient or in more general terms the differential Mueller matrix [1]. A major difficulty in the external calibration of distributed targets is the lack of known distributed targets, and therefore the calibration coefficient must be inferred from point calibration targets. This process is rather complex, particularly when the radar distortions vary over the illuminated area. The distortion parameters for imaging radars can be assumed to be invariant over narrow strips (less than 100 pixels) in the range direction. However, external calibration of imaging radars involves number of other difficulties. For example, in order to obtain the calibration coefficient from known point targets, the ambiguity function of the SAR is needed [2]. Another problem is the issue of signal to background ratio for the point calibration targets. It is required that the radar return in all polarization channels from the calibration target be much larger than the radar return from the surrounding background. This restriction usually puts stringent conditions on the physical size of the calibration targets which in turn introduces uncertainty in the scattering matrix of the target and causes difficulties in target deployment. Furthermore, the interaction of the background surface with the point target introduces yet another uncertainty in the scattering matrix of the target. These interactions include the specular and bistatic scattering from the surface to the point target to the radar and vice versa. To simplify some of these complications, many hybrid calibration techniques based on point calibration targets and assumptions about the statistical properties of distributed targets have been developed [3-4]. Although these methods are rather simple, their accuracy is unknown because of questions about the assumptions made concerning the statistical properties of distributed targets on one hand and the previously mentioned problems associated with point targets on the other.

All these problems could be circumvented provided a standard distributed calibration target existed. In a recent study it was shown that the differential Mueller matrix of distributed targets can be measured very accurately using a polarimetric scatterometer [1]. It is the objective of this paper to develop a rigorous calibration technique for polarimetric SARs using the differential Mueller matrix of a homogeneous distributed target.

MATHEMATICAL FORMULATION

If the scattering matrix of a target is denoted by $\mathbf{S}$, it can be shown that the measured scattering matrix can be obtained from

$$\mathbf{U} = \begin{bmatrix} R_{vv} & R_{vh} \\ R_{hv} & R_{hh} \end{bmatrix} \mathbf{S} \begin{bmatrix} T_{vv} & T_{vh} \\ T_{hv} & T_{hh} \end{bmatrix} . \tag{1}$$

where $\mathbf{R}$ and $\mathbf{T}$ are the receive and transmit distortion matrices respectively. Similarly for a distributed target with radar reflectivity matrix $\mathbf{S}^\circ(x,y)$ the measured scattering matrix is [2]

$$\mathbf{U}(x,y) = \mathbf{R}\left[\int_A \mathbf{S}^\circ(x',y')\,\psi(x-x',y-y')dx'dy'\right]\mathbf{T} , \tag{2}$$

where $\psi(x,y)$ is the ambiguity function of the SAR and A represents an area over which the ambiguity function is non-zero. The reflectivity matrix of the terrain is a random process and so is the measured scattering matrix $\mathbf{U}$. The quantities of interest in radar measurements of distributed targets are the statistical parameters which define the random process as opposed to determination of the sample function ($\mathbf{S}^\circ$) itself. However, determination of the reflectivity matrix from (2) involves a deconvolution process which is extremely difficult if not impossible. In all calibration techniques available in the literature, reconstruction of the actual reflectivity matrix has been attempted without using deconvolution. In other words, calibration algorithms are applied to each individual pixel in the radar image by approximating the original process with another process which is a constant over the illuminated area of the synthesized beam. Then it is hoped that the statistical parameters of the approximate process are identical to those of the original process.

The equivalent scattering matrix of a pixel located at point (x,y) in the image can be defined from (2) and is given by

$$\mathbf{S}(x,y) = \int_A \mathbf{S}^\circ(x',y')\,\psi(x-x',y-y')dx'dy' .$$

Using this definition, (2) can be written as

$$\overline{\mathcal{U}}(x,y) = \mathbf{D}\overline{\mathcal{S}}(x,y) \quad , \tag{3}$$

where $\overline{\mathcal{U}}$ and $\overline{\mathcal{S}}$ are the vector representation of **U** and **S** matrices respectively. The general distortion matrix **D** in terms of the elements of **R** and **T** is found to be

$$\mathbf{D} = \begin{bmatrix} R_{vv}T_{vv} & R_{vv}T_{vh} & R_{vh}T_{vv} & R_{vh}T_{vh} \\ R_{vv}T_{hv} & R_{vv}T_{hh} & R_{vh}T_{hv} & R_{vh}T_{hh} \\ R_{hv}T_{vv} & R_{hv}T_{vh} & R_{hh}T_{vv} & R_{hh}T_{vh} \\ R_{hv}T_{hv} & R_{hv}T_{hh} & R_{hh}T_{hv} & R_{hh}T_{hh} \end{bmatrix}$$

For distributed targets the statistics of the reflectivity process can be obtained from the differential Mueller matrix. Evaluation of the Mueller matrix requires computation of the ensemble average of the cross product of the scattering matrix elements. Using (3) it can be shown that

$$< \mathcal{U}_m\mathcal{U}_n^* > = \sum_{i=1}^{4}\sum_{j=1}^{4} D_{mi}D_{nj}^* < \mathcal{S}_i\mathcal{S}_j^* > \quad .$$

If the random process $\mathbf{S}^\circ$ is stationary and if the reflectivity of every two scattering points within a pixel are uncorrelated, then the autocorrelation function of the process can be written as

$$< \mathcal{S}_i^\circ(x',y')\,\mathcal{S}_j^{\circ *}(x'',y'') > = < \mathcal{S}_i^\circ\mathcal{S}_j^{\circ *} > \delta(x'-x'')\delta(y'-y'') \quad .$$

This assumption is consistent with having many scattering points within each pixel. The differential Mueller matrix in terms of the ensemble average of the cross products is given by

$$\mathbf{M}^\circ = 4\pi\nu\mathbf{W}^\circ\nu^{-1} \quad ,$$

where $\mathbf{W}^\circ$ is the differential covariance matrix and

$$\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -i & i \end{pmatrix} \quad .$$

Using the above autocorrelation function in (3) renders

$$< \mathcal{U}_m\mathcal{U}_n^* > = \left[\int_A |\psi(x-x',y-y')|^2 dx'dy'\right] \sum_{i,j=1}^{4} D_{mi}D_{nj}^* < \mathcal{S}_i^\circ\mathcal{S}_j^{\circ *} > \tag{4}$$

where the integral in the bracket can be regarded as the illumination integral of the synthesized radar beam. Equation (4) establishes the relationship between the covariance of the scattering matrix elements measured by the SAR to the elements of the actual differential covariance matrix of the scene.

DISTORTION PARAMETER RETRIEVAL ALGORITHM

In this section we study equation (4) and examine possible methods of calculating the distortion parameters of the SAR using a distributed target with a known covariance matrix. Equation (4) in matrix notation has the following form

$$\mathbf{V} = I\mathbf{D}\mathbf{W}^\circ\tilde{\mathbf{D}} \quad , \tag{5}$$

where **V** represents the measured covariance matrix and I is the illumination integral defined earlier. Also $\tilde{\mathbf{D}}$ is the conjugate transpose of the **D** matrix. There are a total of eight complex unknowns, however we do not need to know all the unknowns in order to calibrate the measured covariance matrix. In fact (5) can be simplified by factoring out the product $R_{vv}T_{vv}$ from the general distortion matrix **D** which reduces the number of unknowns to six complex and one real quantities of the following form

$$\mathbf{V} = \omega \begin{bmatrix} 1 & x_4 & x_1 & x_1x_4 \\ x_5 & x_6 & x_1x_5 & x_1x_6 \\ x_2 & x_2x_4 & x_3 & x_3x_4 \\ x_2x_5 & x_2x_6 & x_3x_5 & x_3x_6 \end{bmatrix} \mathbf{W}^\circ \begin{bmatrix} 1 & x_5^* & x_2^* & x_2^*x_5^* \\ x_4^* & x_6^* & x_2^*x_4^* & x_2^*x_6^* \\ x_1^* & x_1^*x_5^* & x_3^* & x_3^*x_5^* \\ x_1^*x_4^* & x_1^*x_6^* & x_3^*x_4^* & x_3^*x_6^* \end{bmatrix}$$

where the radiometric calibration constant ω (real quantity) is given by

$$\omega = |R_{vv}T_{vv}|^2 \int_A |\psi(x',y')|^2 dx'dy' \quad .$$

and

$$x_1 = \frac{R_{vh}}{R_{vv}}, \quad x_2 = \frac{R_{hv}}{R_{vv}}, \quad x_3 = \frac{R_{hh}}{R_{vv}},$$
$$x_4 = \frac{T_{vh}}{T_{vv}}, \quad x_5 = \frac{T_{hv}}{T_{vv}}, \quad x_6 = \frac{T_{hh}}{T_{vv}}.$$

Equation (5) provides six complex and four real independent nonlinear equations which can be solved by various numerical techniques. However, the ability of numerical methods to find the solutions to simultaneous equations depends on how close the initial guess is to the solution. It is usually very difficult to characterize the behavior of a multi-valued function with a domain in a large vector space. In this paper instead of following a brute force numerical method in finding the solution, we isolated one of the variables into an equation using properties of similar matrices.

NUMERICAL RESULTS AND EXAMPLES

To assess the accuracy and validity of the model developed in previous sections, the backscatter data collected in the cross calibration experiment of July 1991 will be used [10]. In this experiment, the JPL AIRSAR and truck-mounted scatterometers were used to measure the backscatter from four different homogeneous distributed targets at the same incidence and look angles. Three rough surfaces with rms height and correlation length $s = 0.78cm$ $L = 10.5cm$, $s = 1.2cm$ $L = 9cm$, and $s = 4cm$ $L = 15.2cm$ respectively were generated using farming equipment each having dimension $100m \times 300m$. The fourth target was a hay field with vegetation height of $50cm$ and vegetation biomass of $667g/m^2$.

For an incidence angle of 30° and L-band SAR, we used the hay field as the calibration target and after application of the algorithm outlined in Section 3, the distortion parameters of the L-band SAR were found to be

$$\omega = 18.2dB \quad ,$$
$$\frac{R_{vh}}{R_{vv}} = -19.8dB\angle 33^\circ \quad , \quad \frac{R_{hv}}{R_{vv}} = -23.0dB\angle 36.7^\circ \quad ,$$
$$\frac{R_{hh}}{R_{vv}} = 0.6dB\angle -10.7^\circ \quad , \quad \frac{T_{vh}}{T_{vv}} = -17.7dB\angle 37.3^\circ \quad ,$$
$$\frac{T_{hv}}{T_{vv}} = -18.4dB\angle 57.9^\circ \quad , \quad \frac{T_{hh}}{T_{vv}} = 0.0dB\angle -7.6^\circ \quad .$$

Using these values, Fig. 1 shows comparison between the scatterometer and SAR measurement of backscattering coefficients of distributed targets. Considering that the uncertainty in the scatterometer measurement is about $\pm 0.5dB$, the agreement between the SAR and scatterometer measurement is better than $\pm 1dB$. For C-band SAR we used the medium rough surface as the calibration target and the distortion parameters are as follow

$$\omega = 22.3dB \quad ,$$
$$\frac{R_{vh}}{R_{vv}} = -10.5dB\angle 22.8^\circ \quad , \quad \frac{R_{hv}}{R_{vv}} = -73.2dB\angle 23.4^\circ \quad ,$$
$$\frac{R_{hh}}{R_{vv}} = -0.2dB\angle 49.8^\circ \quad , \quad \frac{T_{vh}}{T_{vv}} = -21.2dB\angle 2.1^\circ \quad ,$$
$$\frac{T_{hv}}{T_{vv}} = -12.3dB\angle 32.4^\circ \quad , \quad \frac{T_{hh}}{T_{vv}} = -1.1dB\angle 10.2^\circ \quad .$$

Figure 2 shows the comparison between the C-band SAR and scatterometer measurements. Again the agreement between the two system measurements is better than $\pm 1dB$. To complete the accuracy assessment, we next consider the comparison in phase difference statistics. In a recent study it was shown that the p.d.f. of the co- and cross-polarized phase differences can be obtained from the Mueller matrix [5]. The p.d.f. is completely characterized by two parameters, degree of correlation (α), which is a measure of the distribution width and the polarized phase difference (ζ), which represents the phase difference at the p.d.f. maxima. Figures 3 and 4 show the comparison between the SAR and the scatterometers for the co- and cross-polarized degree of correlation for L- and C-band respectively. Figure 5 compares the polarized phase difference measured by SAR and scatterometer at L- and C-band for co-polarized channels. Considering the fact that phase measurement with scatterometers has an uncertainty of $\pm 5°$ the agreement between the two systems is better than $\pm 10°$.

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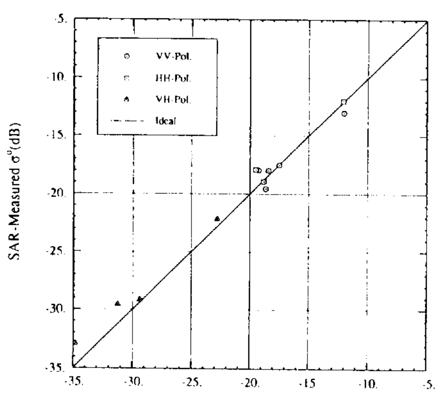

Figure 1: Comparison between the scatterometer and SAR measurement of backscattering coefficients of distributed targets at L-band.

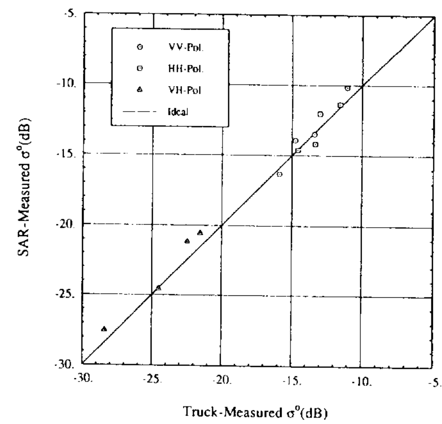

Figure 2: Comparison between the scatterometer and SAR measurement of backscattering coefficients of distributed targets at C-band.

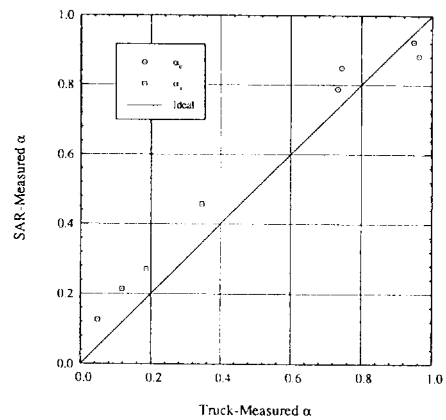

Figure 3: Comparison between the SAR and the scatterometer measurement of the co- and cross-polarized degree of correlation at L-band.

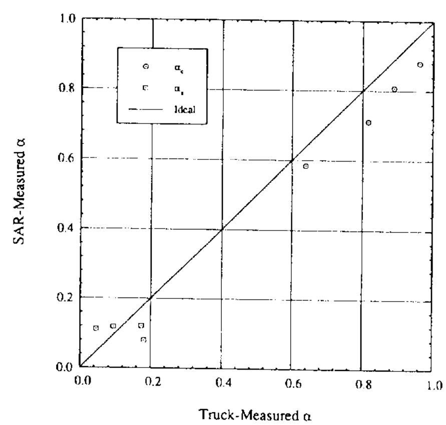

Figure 4: Comparison between the SAR and the scatterometer measurement of the co- and cross-polarized degree of correlation at C-band.