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Nanotechnology: An Introduction

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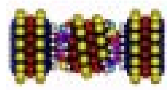
Outline

- Introduction
- Nanotechnology Examples
- Quantum Computing



What's Nano?

- Micro (as in microelectronics) = Small
- Nano (as in nanoelectronics) = *Really* small
- 1 nanometer = 10^{-9} meters
  = width of a few atoms
- **Nanotechnology**: Building things such as electrical, mechanical and biological devices atom by atom

What's a Nanometer?

Ref: Morrison and Morrison: *Powers of Ten*, 1994

Pre-History

Richard P. Feynman
(1918-1988)

- " The principles of physics, as far as I can see, do not speak against the possibility of maneuvering things atom by atom. It is not an attempt to violate any laws… it has not been done because we are too big." [*There's Plenty of Room at the Bottom*, 1959]
- "At any rate, it seems that the laws of physics present no barrier to reducing the size of computers until bits are the size of atoms and quantum behavior holds dominant sway." [*Quantum Mechanical Computers*, 1985]

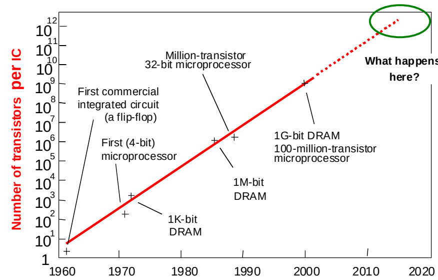
Why the Interest?

- Invention of atomic manipulation tools in 1980s
- Remarkable scientific discoveries in 1990s
  - Carbon nanotubes
  - Biomolecular machines
  - Molecular circuits
  - DNA computing
  - Quantum computing
  - etc.
- U.S. National Nanotechnology Initiative (2000)

Why the Interest (contd)?

- Forecasts of the end of Moore's Law
- Nanoscale implies
  - > Higher density
  - > Lower power
  - > Higher speed
- Nanoelectronics
  - > Single electron transistors
  - > Molecular circuits
  - > Spintronics
  - > etc.

Moore's Law



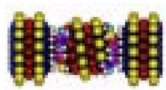
Atomic Manipulation Tools

- Molecular beam epitaxy

- Scanning probe microscopes
- Molecular self-assembly

Outline

- Introduction
- **Nanotechnology Examples**
- Quantum Computing

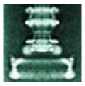


Nanotechnology Examples

Writing Atom by Atom

Nanotechnology Examples

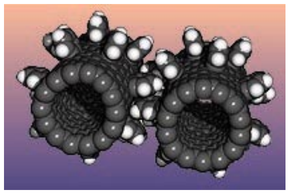
Biological Nanomotor



Ref: *Sci.American*, Sept. 2001

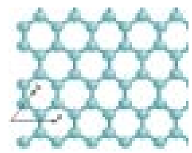
Nanotechnology Examples

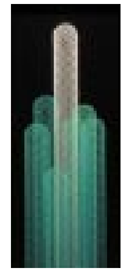
Atomic Gear Wheels (NASA)



Nanotechnology Examples

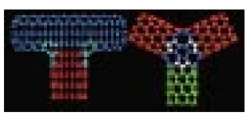
Carbon Nanotubes





Carbon Nanotubes

- Made by folding a graphene sheet into a tube a few nanometers in diameter
- Depending on how it's rolled up, a nanotube can be a metal or a semiconductor
- Along its axis a nanotube is as stiff as diamond
- Metallic nanotubes make good wires
- Nanotube junctions can, in principle, make transistors



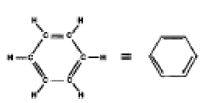
Molecular Circuits

- Wires and switches can be formed from organic molecules and synthesized into logic circuits
- Wires have relatively low conductivity
- Diode switches can be built
- Transistors or other devices with signal gain present a major challenge

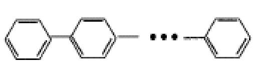
Molecular Circuits

Wires

Benzene C_6H_6

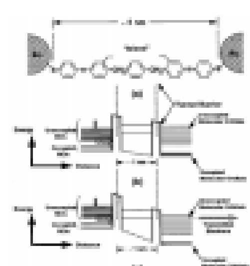


Polyphenylene chain



Molecular Circuits

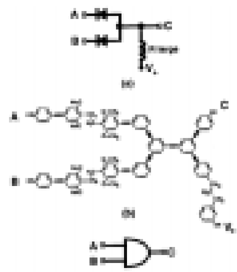
Resonant Tunneling Diode



Ref: Ellenbogen & Love: *Proc. IEEE*, Mar. 2000

Molecular Circuits

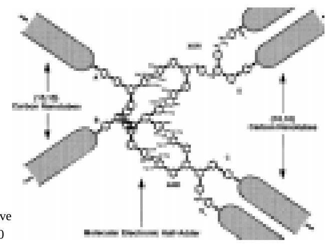
AND Gate



Ref: Ellenbogen & Love: *Proc. IEEE*, Mar. 2000

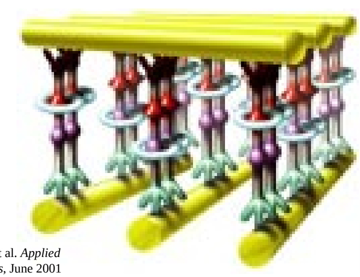
Molecular Circuits

Half Adder



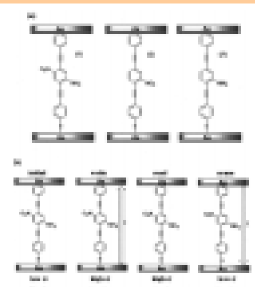
Ref: Ellenbogen & Love *Proc. IEEE*, Mar. 2000

Molecular RAM



Ref: Reed et al. *Applied Phys. Letters*, June 2001

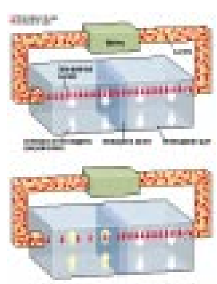
Molecular RAM



Ref: Reed et al. *Applied Phys. Letters*, June 2001

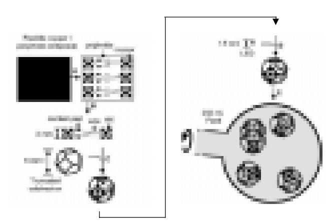
“Spintronics”

Magnetic Tunnel Junction



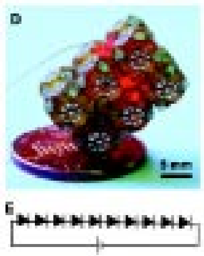
Ref: Zorpette. *IEEE Spectrum*, Dec. 2001

3D Self-Assembled Circuits



Ref: Gracias, Whitesides et al. *Science*, Aug. 2000

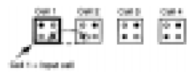
3D Self-Assembled Circuits



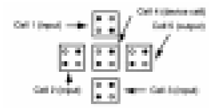
Ref: Gracias, Whitesides et al. *Science*, Aug. 2000

Quantum-Dot Circuits

Quantum Cellular Automata (QCA)
[Univ. of Notre Dame]

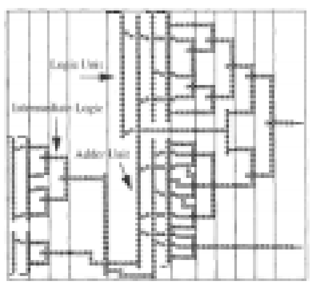


QCA "wire"



QCA majority gate

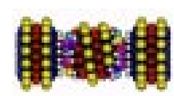
Quantum-Dot Circuits



QCA ALU (partial)

Outline

- Introduction
- Nanotechnology Examples
- **Quantum Computing**

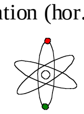


Computational Limits

- **Question:** Is there a different and fundamentally faster way to compute?
- **Answer:** Yes – maybe!
  Quantum mechanics can form the basis for an entirely new type of computation—quantum computing — if some huge practical implementation problems can be solved

Quantum Information

- Information is stored in a quantum state, e.g. photon polarization (hor./vert.) or electron spin (up/down)



"Zero" state denoted $|0\rangle$

"One" state denoted $|1\rangle$

- The quantum state is a superposition of the zero and one states called a **qubit**

$$c_0|0\rangle + c_1|1\rangle$$

Quantum Information

- **The Good News**
  - > N qubits can store 2^N binary numbers simultaneously, suggesting massive parallelism

    $N = 2$: $|\Psi\rangle = c_0|00\rangle + c_1|01\rangle + c_2|10\rangle + c_3|11\rangle$

  or, in general,

  $$|\Psi\rangle = \sum_{i=0}^{2^n-1} c_i \left| b_{i,n-1} b_{i,n-2} \ldots b_{i,0} \right\rangle$$

  - > Quantum states have wavelike properties that allow powerful non-classical interactions (interference, entanglement)

Quantum Information

- **The Bad News**
  - > Measurement yields just one of the 2^N superimposed numbers $|b_{i,n-1}\, b_{i,\,n-2} \ldots b_{i,0}\rangle$ and destroys the superposition
  - > The measured results are random: the measured state is $|b_{i,n-1}\, b_{i,\,n-2} \ldots b_{i,0}\rangle$ with probability $|c_i|^2$
  - > Quantum states are very fragile due to interaction with the environment (decoherence)
  - > Quantum computing devices are very hard to build using existing (nano)technologies

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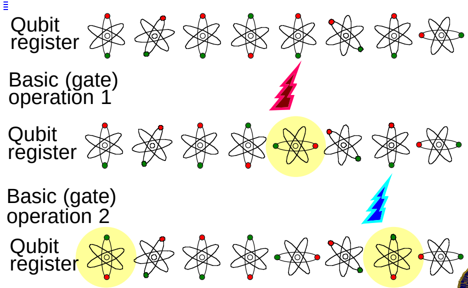
Quantum Operations



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History

- **1982**: Richard Feynman suggested quantum mechanics could provide an exponential speed-up in simulation
- **1985**: David Deutsch described a simple algorithm exhibiting quantum parallelism
- **1994**: Peter Shor showed how to factor integers into primes in polynomial time using quantum methods, thus "breaking" RSA encryption
- **1996-98**: Demonstration quantum computing devices built at LANL, Oxford, etc. employing a few (≤ 10) qubits

  Quantum communication successfully demonstrated over long distances

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Classical Logic Circuits

- Behavior is governed implicitly by classical physics: no restrictions on copying or measuring signals
- Signal states are simple bit vectors, e.g. $X = 01010111$
- Signal operations are defined by Boolean algebra
- Small well-defined sets of universal gate types exist , e.g. {NAND}, {AND, OR, NOT}
- Circuits use fast, scalable and macroscopic technologies such as transistor-based CMOS integrated circuits

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Quantum Circuits

- Behavior is governed by quantum mechanics
- Signal states are qubit vectors
- Operations are defined by linear algebra over Hilbert space and can be represented by "unitary" matrices
  - > Gates and circuits must be reversible (information-lossless)
  - > Number of output lines = Number of input lines
- Many universal gate sets exist but the best types are not obvious

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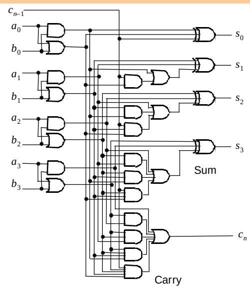
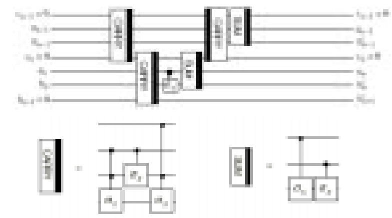
Classical vs. Quantum Circuits

Classical adder

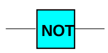


Classical vs. Quantum Circuits

Quantum adder



Quantum Gates

- **One-Input gate:** NOT
  - Input state: $c_0|0\rangle + c_1|1\rangle$
  - Output state: $c_1|0\rangle + c_0|1\rangle$
  - Graphic symbol for NOT: 

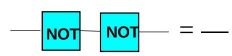
  - "Pure" states $|0\rangle$ and $|1\rangle$ are mapped thus:

$$|0\rangle \rightarrow |1\rangle$$
$$|1\rangle \rightarrow |0\rangle$$

Quantum Gates

- **NOT gate** (contd.)
  - Vector notation for states: $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
  - Matrix notation for gate operation: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
  - Gate connection corresponds to matrix multiplication:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

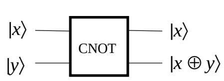


Identity matrix

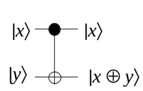
NOT matrix

Quantum Gates

- **Two-Input Gate:** Controlled NOT (CNOT)



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



  - CNOT maps

$$|x\rangle|0\rangle \rightarrow |x\rangle||x\rangle$$

and

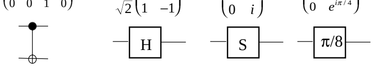
$$|x\rangle|1\rangle \rightarrow |x\rangle||\text{NOT}(x)\rangle$$

Quantum Gates

Discrete Universal Gate Set

- Four-member "standard" gate set

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$



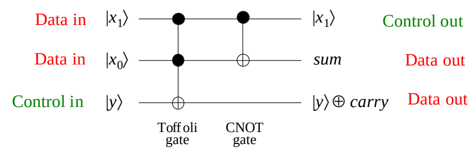
CNOT Hadamard Phase π/8 (T) gate

Quantum Circuits

- A quantum (combinational) circuit is a sequence of quantum gates
- The circuit has fixed "width" corresponding to the number of qubits being processed
- Logic design (classical and quantum) attempts to find circuit structures for needed operations that are
  - Functionally correct
  - Independent of physical technology
  - Low-cost, e.g. use the minimum number of qubits or gates

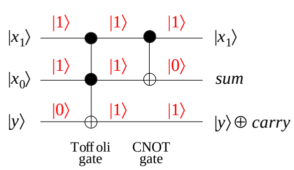
Quantum Circuits

- **Example 1: Quantum Half Adder**
  - Compute the sum and carry for two qubits x_1, x_0



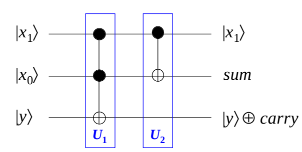
Quantum Circuits

- **Quantum Half Adder:** Simulation
  - Verify the add truth table



Quantum Circuits

- **Quantum Half Adder:** Algebraic analysis
  - Involves an 8-D vector space with basis $|000\rangle, |001\rangle, \ldots, |111\rangle$
  - Adder's behavior corresponds to an 8×8 matrix U
  - What is $U(x_1, x_0, y) = U_2 U_1(x_1, x_0, y)$?



Quantum Circuits

- **Quantum Half Adder:** Algebraic analysis (contd.)
  - Matrix U_1 for the Toffoli gate

$$U_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Quantum Circuits

- **Quantum Half Adder:** Algebraic analysis (contd.)
  - Matrix U_2 for the CNOT gate and "wire" combination

$$U_2 = U_{CNOT} \otimes I_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Tensor product

Quantum Circuits

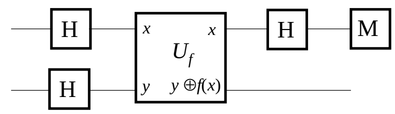
- **Quantum Half Adder:** Algebraic analysis (contd.)
  - Matrix U for the overall circuit

$$U = U_2U_1 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0 \end{pmatrix} = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0 \end{pmatrix}$$

Quantum Circuits

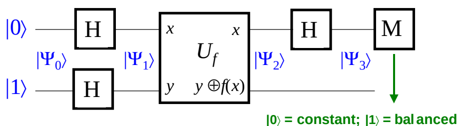
Example 2: Implementing Deutsch's Algorithm

- *Problem:* Determine whether a one-variable Boolean function $f(x)$ is constant, i.e. $f(0) = f(1)$, or balanced, i.e. $f(0) \neq f(1)$.
- Classical algorithms require two evaluations of f.
- This algorithm uses just one quantum evaluation by, in effect, computing $f(0)$ and $f(1)$ simultaneously
- **Circuit**:



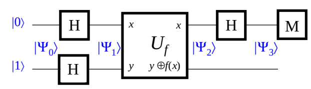
Quantum Circuits

- **Deutsch's Algorithm** (contd.)



- Initialize with $|\Psi_0\rangle = |01\rangle$
- Create superposition of x states using the first Hadamard (H) gate. Set y control input using the second H gate
- Compute $f(x)$ using the special unitary circuit U_f
- Interfere the $|\Psi_2\rangle$ states using the third H gate
- Measure the x qubit

Quantum Circuits



$$|\Psi_0\rangle = [|0\rangle][|1\rangle] \qquad |\Psi_1\rangle = \left[\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right]\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$

$$|\Psi_2\rangle = \left[\frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}}\right]\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] = \begin{cases} \pm\left[\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right]\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] & \text{if } f(0) = f(1) \\ \pm\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] & \text{if } f(0) \neq f(1) \end{cases}$$

$$|\Psi_3\rangle = \begin{cases} \pm[|0\rangle]\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] & \text{if } f(0) = f(1) \\ \pm[|1\rangle]\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] & \text{if } f(0) \neq f(1) \end{cases}$$

Physical Implementation

Main Contenders

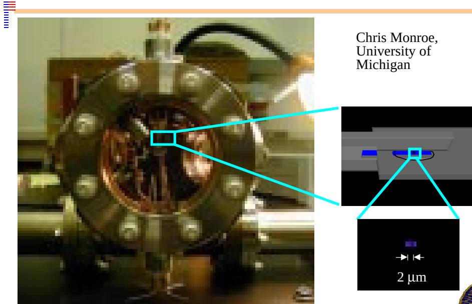
- Nuclear magnetic resonance (NMR)
- Ion traps
- Semiconductor quantum dots
- Optical lattices

  etc.

Main Deficiency

- Poor scalability

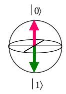
Ion Trap Approach



Chris Monroe, University of Michigan

NMR Approach

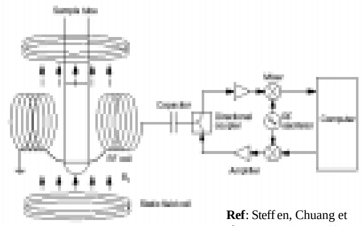
- Many atoms have a nucleus with quantum “spin” like a tiny bar magnet. Spin up/down = $|0\rangle/|1\rangle$.



- Several atoms’ spins can be coupled chemically in a molecule but are individually addressable due to different resonant frequencies
- An RF pulse can rotate an atom’s spin in a manner proportional to the amplitude and duration of the applied pulse
- A gate is a sequence of carefully sized and separated RF pulses
- Many molecules (10^{18}) can be combined in liquid solution to form a same-state ensemble of macroscopic and manageable size
- Scalability seems to be limited to < 100 qubits

NMR Implementation

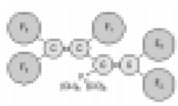
- Five-qubit NMR computer



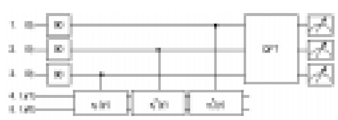
Ref: Steffen, Chuang et al.: *IEEE Micro*, 2001

NMR Implementation

- Five-qubit computer (contd.)
  - Molecule with 5 flourine atoms whose spins implement the qubits



  - Experimental 5-qubit circuit to find the order of a permutation



Summary: Nanotechnology

- **Common theme**: Nanometer-scale devices based on atomic physics phenomena such as electron spin, carbon properties, etc.
- Novel device and circuit *technologies* based on molecules, nanotubes, self-assembly, etc.
- Novel computing and communication *science* based on quantum mechanics
- Current physical technologies are primitive, fragile, and limited to demonstration devices
- There’s a long way to go!