EECS 203-1 – Winter 2002
Definitions review sheet - Midterm 1

- Propositional variable and propositional expression: A *propositional variable* is just a name, like $p, q, \ldots$. A *propositional expression* is either a propositional variable, or a formula in one of the forms $P \wedge Q$, $P \vee Q$, $\neg P$, $P \rightarrow Q$, or $P \leftrightarrow Q$, where P and Q are themselves propositional expressions.

- Definitions of tautology, contradictory formula, satisfiable formula, for PROPOSITIONAL calculus: A propositional expression is a *tautology* if it is true for all possible assignments of truth values to its variables. A *contradictory expression* is false for all assignments of truth values to its variables. A *satisfiable formula* is an expression which is true for at least one assignment.

- Logical equivalence and implication in propositional calculus: Two propositional expressions P and Q are *logically equivalent* if the expression $P \leftrightarrow Q$ is a tautology. We write this as $P \Leftrightarrow Q$. We say that P *logically implies* Q if the expression $P \rightarrow Q$ is a tautology.

- Predicate symbols for first order logic: A *predicate symbol* is an expression of the form $P(x, y, \ldots, a, b, \ldots)$, where P is just a name, and $x, y, \ldots$ are *individual variables*, and $a, b, \ldots$ are *individual constants.*

- A first-order formula in first order logic is either a predicate symbol, or of one of the forms $\phi \wedge \psi, \phi \vee \psi, \neg\phi, \phi \rightarrow \psi, \phi \leftrightarrow \psi$, where ϕ and ψ are first-order formulas, or else is of the form $\forall x\phi$ or $\exists x\phi$, where x is an individual variable, and ϕ is a first-order formula.

- A sentence, in first order logic, is a formula with no free variables. Sentences are the only formulas which can be true or false. To be true or false, a sentence needs a world.

- A world (context, universe) consists of a collection of objects, and some specific properties and relations. An interpretation of a formula in a world consists of matching the predicate symbols in the formula with the actual properties and relations in the world.

- Logical equivalence; universally valid sentences; logical implication, in FIRST ORDER logic: A sentence is *universally valid* if it is true in all worlds. We say that a sentence ϕ *logically implies* a sentence ψ ($\phi \Rightarrow \psi$), if $\phi \rightarrow \psi$ is logically valid. Finally, a sentence ϕ is *logically equivalent* to ψ ($\phi \iff \psi$), if $\phi \leftrightarrow \psi$ is logically valid.

- A set is just a group or collection of objects – no formal definition. However, we can specify sets by listing or by using set-builder notation. We can also have variables like X, Y that range over sets. There is also a relation "element-of" between objects and sets. We write $x \in A$ and say "x is an element of A".

- Definition of subset relation using "element of" relation: We define $A \subseteq B$ (A is a subset of B) if

  $$\forall x(x \in A \to x \in B).$$

  Two sets are equal if each is a subset of the other.

- Definitions of union, intersection, complement: Given sets A and B, both subsets of some universe U, we have the *union* of A and B,

  $$A \cup B = \{x \mid x \in A \vee x \in B\}.$$

  The *intersection* of A and B is

  $$A \cap B = \{x \mid x \in A \wedge x \in B\}.$$

  The *complement* $\overline{A}$ of A is $\{x \in U \mid x \notin A\}$.
  The *set difference* $A \setminus B$ is $A \cap \overline{B}$.

- Power set: $\mathcal{P}(A) = \{X \mid X \subseteq A\}$.

- Cartesian product: $A \times B = \{(a, b) \mid a \in A, b \in B\}$.