EECS 203-1: sample midterm

All problems are worth 15 points each. Total: 60 points. Show all work; the exam is CLOSED BOOK, CLOSED NOTES. Good luck!

- 1. (Definitions) Give precise definitions of the following notions.
- the powerset $\mathcal{P}(S)$ of a set S;
- the Cartesian product of two sets A and B;
- the subset relation between sets A and B.

2. For each of the following pairs (E_1, E_2) of expressions, say whether or not (a) E_1 is logically equivalent to E_2 ; (b) E_1 logically implies E_2 ; or (c) neither. If you answer (c), then show an example why E_1 does not imply E_2 .

•
$$(P \land (Q \to P), Q)$$
•
$$((P \to Q) \land \neg Q, \neg P)$$
•
$$(\neg (P \lor Q), \neg P \land \neg Q)$$

3. (Translations) With universe of discourse the natural numbers $\{0, 1, 2, ...\}$, let Prime(n) mean "n is prime" and Odd(n) mean "n is odd". You may also use the "=" predicate.

(a) Translate the following into logical notation.

- There are two prime numbers whose sum is odd.
- Whenever the sum of two prime numbers is odd, neither of them equals 2.
- Any two prime integers have a sum which is even.

(b.) Write the following in English:

$$(\forall n)(Odd(n) \rightarrow (\exists m)(Prime(m+n)))$$

4. Prove that for sets A, B, C, that if $A \oplus B = A \oplus C$ then B = C. Recall that $X \oplus Y = (X \cap \overline{Y}) \cup (Y \cap \overline{X})$.