EECS 203-1 - Homework 3
Exercises in Rosen,
page 34-37: 12 bdfh, 14 acegi, 34, 38, 40, 52
page 37: 40 again
page 45: 4, 6, 12 ad, 14

Total points = 60

12 (b) (2pts)
$\forall y F(\text{Evelyn}, y)$

(d) (2pts)
$\neg\exists x \forall y F(x, y)$
$= \forall x(\neg\forall y F(x, y))$
$= \forall x \exists y(\neg F(x, y))$

(f) (3pts)
$\neg\exists x(F(x, \text{Fred}) \wedge F(x, \text{Jerry}))$
$= \forall x(\neg(F(x, \text{Fred}) \wedge F(x, \text{Jerry})))$
$= \forall x(\neg F(x, \text{Fred}) \vee \neg F(x, \text{Jerry}))$

(h) (3pts)
$\forall x \exists y \forall z(F(x, y) \wedge ((y \neq z) \rightarrow \neg F(x, z))$

14 (2pts each)

(a) $\neg I(\text{Jerry})$

(c) $\neg C(\text{Jan}, \text{Sharon})$

(e) $\forall x((x \neq \text{Joseph}) \leftrightarrow C(\text{Sanjay}, x))$

(g) $\neg\forall I(x)$
$= \exists x \neg I(x)$

(i) $\forall x \exists y((x \neq y) \leftrightarrow I(x))$

34 (2pts each)

(a) $\neg\exists x(P(x) \wedge S(x))$
$= \forall x \neg(P(x) \wedge S(x))$
$= \forall x(\neg P(x) \vee \neg S(x))$
$= \forall x(P(x) \rightarrow \neg S(x))$

(b) $\neg\exists x(R(x) \wedge \neg S(x))$
$= \forall x \neg(R(x) \wedge \neg S(x))$
$= \forall x(\neg R(x) \vee S(x))$
$= \forall x(P(x) \rightarrow S(x))$

(c) $\forall x(Q(x) \rightarrow P(x))$

(d) $\forall x(Q(x) \rightarrow \neg R(x))$

(e) Yes.

38 (3pts each)

(a) Suppose A is TRUE:
LHS = TRUE
RHS = TRUE
Suppose A is FALSE:
LHS = $\forall x P(x)$
RHS = $\forall x P(x)$
Therefore, logical equivalence.

(b) Suppose A is TRUE:
LHS = TRUE
RHS = $\exists x$(TRUE) = TRUE
Suppose A is FALSE:
LHS = $\exists x P(x)$
RHS = $\exists x P(x)$
Therefore, logical equivalence.

40 (4pts)

We prove that the two are not logically equivalent by using a counter example.

Let $P(x)$ be "x is a positive number" and let $Q(x)$ be "x is a non-positive number" and the universe of discourse is the set of all real numbers.

Then $\forall x P(x) \vee \forall x Q(x)$ is FALSE. But $\forall x(P(x) \vee Q(x))$ is TRUE.

52 (3pts)

$\forall \epsilon \exists N(n > N \rightarrow \mid a_n - L \mid < \epsilon)$, for $\epsilon > 0$ and $N > 0$

40 again (3pts)

$(\forall x P(x) \vee \forall x Q(x))$ implies $\forall x(P(x) \vee Q(x))$

4 (4pts)

$B \subset A$
$C \subset A$
$C \subset D$

6 (4pts)

{2} is an element of the sets of 5c, 5d and 5e, but not others.

12 (2pts each)

(a) cardinality=0.

(d) cardinality=3.

14 (2pts)

Yes.