EECS 203-1 Homework –8 Solutions (modified 03/21/02)

Total Points: 40
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4) Determine whether the relation *R* on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a,b) \in R$ if and only if

4 points

a) *a* is taller than *b*
Transitive and vacuously Anti-symmetric (nobody is taller than himself

b) *a* and *b* were born on the same day
Reflexive, Symmetric and Transitive

c) *a* has the same first name as *b*
Reflexive, Symmetric and Transitive

d) *a* and *b* have a common grandparent
Reflexive and Symmetric

18) Let $R_1 = \{(1,2), (2,3), (3,4)\}$ and $R_2 = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (3,4)\}$. Find

4 points

a) $R_1 \cup R_2$
$\{(1,2), (2,3), (3,4), (1,1), (2,1), (2,2), (3,1), (3,2), (3,3)\}$

b) $R_1 \cap R_2$
$\{(1,2), (2,3), (3,4)\}$

c) $R_1 - R_2$
$\{\varnothing\}$

d) $R_2 - R_1$
$\{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3)\}$

28b) Suppose that R and S are reflexive relations on a set A. Prove or Disprove each of the following statements.

2 points

b) $R \cap S$ is Reflexive

Proof: $R \cap S$ is reflexive means that $(a,a) \in R \cap S$ for every element a of set A.
Since R and S are reflexive, then $(a,a) \in R$ and $(a,a) \in S$ for every element of A.
Therefore (a,a) must be in their intersection.

36) Let R be a symmetric relation. Show that R^n is symmetric for all positive integers n.

5 points

Let R be a symmetric relation on set A

Proof by induction:

Basis Step: $R^1 = R$ is symmetric is True.

Inductive Step: Assume that R^n is symmetric.

To Prove that R^{n+1} is symmetric.

R^{n+1} is symmetric if for all (x,y) in R^{n+1}, we have (y,x) is in R^{n+1} as well.

Assume that (x,y) is in R^{n+1}.

Now, $R^{n+1} = R^n \mathbf{o} R = R \mathbf{o} R^n$

We know that if $(x,y) \in R \mathbf{o} R^n$, then by the definition of composition there exists a z in A such that xRz and $z(R^n)y$ i.e (x,z) is in R and (z,y) is in R^n

And we also know that R and R^n are symmetric, which implies that (z,x) is in R and also (y,z) is in R^n.

Therefore, by definition of composition,

$(y,x) \in R \mathbf{o} R^n$; i.e.; $(y,x) \in R^{n+1}$.

Hence Proved.

Does a relation on a set A, which is symmetric and transitive necessarily have to be reflexive? Prove it or give a counterexample relation.

2 points

Counter Example Relation:

Let A={a, b, c}

Let R be a relation on set A

R ={(a, a), (b, b), (a, b), (b, a)}

This relation is symmetric and transitive but not reflexive since it doesn't contain (c,c).

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2) List the ordered pairs in the relations on {1,2,3} corresponding to the following matrices (where rows and columns correspond to the integers listed in increasing order).

3 points

a)

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

{(1,1), (1,3), (2,2), (3,1), (3,3)}

b)

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

c) **{(1,2), (2,2), (3,2)}**

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

{(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3) }

8) Let R_1 and R_2 be relations on a set A represented by the matrices.

$$\mathbf{M}_{R1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad \mathbf{M}_{R2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the Matrices that represent

6 points

a) $R_1 \cup R_2$

$$\begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix}$$

b) $R_1 \cap R_2$

$$\begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

c) $R_2 \circ R_1$

$$\begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

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26) Use Algorithm 1 to find the transitive closures of the following relations on {a, b, c, d, e}.

8 points

a) {(a,c), (b,d), (c,a), (d,b), (e,d)}

$$B = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

b) {(b,c), (b,e), (c,e), (d,a), (e,b), (e,c) }

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Recall the recursive definition of transitive closure:
Given a binary relation R on a set A, we use the following rules to construct the relation tc(R).
1. "Basis": if $(x, y) \in R$, then $(x, y) \in tc(R)$.
2. "Induction": if $(x, y) \in tc(R)$ and $(y, z) \in tc(R)$, then $(x, z) \in tc(R)$.
3. No pair is in tc(R) unless it is shown there using a finite number of applications of rules 1 and 2.
Instead, prove by induction on n that if (x,y) is in R^n, then (x; y) can be shown to be in tc(R)using some finite number of rule applications.

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6 points

Basis step: for n=1, if $(x, z) \in R$, then (x, z) gets into tc(R) by 1 rule application.
Inductive Step: Assume that if $(x, z) \in R^n$, then $(x, z) \in tc(R)$ (which by the construction of tc(R) means it got in by finite number of rule applications.
To Prove that if $(x, z) \in R^{n+1}$, then (x, z) gets into tc(R) by *n+1* or fewer rule applications.
Now, $R^{n+1} = R^n \mathbf{o} R = R \mathbf{o} R^n$.
If $(x,z) \in R^n \mathbf{o} R$, then by the definition of composition there exists a y in A such that (x,y) is in R^n and (y,z) is in R. By induction hypothesis, (x,y) is in tc(R) in finite number of rule applications and if (y,z) is in R then it is tc(R) by the application of rule 1. Now one more application of rule 2 tells us that (x,z) is in tc(R) in finite number of rule applications. Therefore P(n) is true implies P(n+1) is true.
Thus proved.