Quiz 1
EECS 203
Spring 2016

Name: ______________________________________ uname: __________________________

Instructions.

You have 25 minutes to complete this quiz. You may not use any sources of information, including electronic devices, textbooks, or notes. Leave at least one seat between yourself and other students. Please write clearly. If we cannot read your writing, it will not be graded. Do not write answers on the back pages of the exam. If you finish early, you are welcome to get up and turn in the exam, but please do so quietly. We expect class will start around 10:45am.

Honor Code.

This course operates under the rules of the College of Engineering Honor Code. Your signature endorses the pledge below. ***After*** you finish your exam, please sign on the line below:

I have neither given nor received aid on this examination, nor have I concealed any violations of the Honor Code.

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1) **(5 points, -2 per incorrect circle/non-circle, min 0 points).** We define the following propositions

- G: I like grape juice
- E: Grape juice is expensive
- B: I will buy grape juice.

Consider the sentence "If I don't like grape juice and it is expensive, I will not buy grape juice". Zero or more of the following propositions correctly describe the sentence. Circle each statement that is logically equivalent to the sentence.

a) $(G \vee \neg E) \to B$

b) $B \to (G \vee \neg E)$

c) $\neg B \vee (\neg G \wedge E)$

d) $\neg B \wedge (\neg G \wedge E)$

e) $(\neg G \wedge E) \to \neg B$

Answer: b,e **e is the direct translation, and b is its contrapositive**

2) **(8 points, -2 per wrong/blank answer).** For each of the following circle an answer indicating if the proposition is either a tautology, a contradiction, or a contingency.

a) $(a \wedge b) \to a$ **tautology - this is a statement of the law of simplification**

b) $(a \vee b) \to (a \wedge b)$ **contingency - TT makes it true, while TF makes it false**

c) $(a \oplus b) \to (a \vee b)$ **tautology - they are only different when ab=FF, which makes it F->T=T**

d) $(a \wedge b) \leftrightarrow (a \oplus b)$ **contingency - TT makes it false, while FF makes it true**

e) $(a \vee b) \leftrightarrow (a \oplus b)$ **contingency - TT makes it false, while FF makes it true**

f) $(a \vee b) \wedge (a \vee \neg b) \wedge (\neg a \vee b) \wedge (\neg a \vee \neg b)$ **contradiction - each assignment makes 1 term false**

3) **(5 points, -2 per wrong/blank answer)** We define the following predicates

- **C(x,y):** y is the child of x
- **W(x):** x works at a grocery store

Consider the statement "There is at least one person which has all of their children working at a grocery store." Zero or more of the expressions below are accurate translations of this statement. Circle each of the following that are correct.

a) $\exists x \forall y \, (C(x,y) \wedge W(y))$

b) $\exists x \forall y \, (C(x,y) \to W(y))$

c) $\forall y \exists x (C(x,y) \wedge W(y))$

d) $\neg \forall x \forall y (C(x,y) \to W(y))$

e) $\exists x \forall y \, (\neg C(x,y) \vee W(y))$

Answer: b,e. b is the direct translation, while e expands the definition of implication

4) **(9 points)** Prove or disprove the following using any method. You must clearly show your work.

$((p \lor q) \to \neg a) \equiv (a \to (\neg q \land \neg p))$

Logical equivalence:

$(p \lor q) \to \neg a \equiv a \to \neg(p \lor q)$	Contrapositive
$\equiv a \to (\neg p \land \neg q)$	De Morgan's Law
$\equiv a \to (\neg q \land \neg p)$	Commutative

Truth table:

p	q	a	$\neg p$	$\neg q$	$\neg a$	$p \lor q$	$(p \lor q) \to \neg a$	$\neg p \land \neg q$	$a \to (\neg p \land \neg q)$
F	F	F	T	T	T	F	T	T	T
F	F	T	T	T	F	F	T	T	T
F	T	F	T	F	T	T	T	F	T
F	T	T	T	F	F	T	F	F	F
T	F	F	F	T	T	T	T	F	T
T	F	T	F	T	F	T	F	F	F
T	T	F	F	F	T	T	T	F	T
T	T	T	F	F	F	T	F	F	F

Because the last and third to last columns are the same, these expressions are equivalent.

5) **(3 points, -1 per wrong/blank answer)** Circle each of the listed domains for which the following statement is true: $\forall x \exists y \, (x^2 > y)$

a) Positive integers (so not including zero as zero is not positive)

b) Integers

c) Reals

d) Non-negative integers

e) **Positive Reals**

Answer: b, c, e. For both b and c, y=-1 works for any x. For e, $y = \sqrt{x}/2$ **is still positive real**