Quiz 2

EECS 203

Spring 2016

Name (Print):_____Answer____uniqname (Print): ___Key________

Instructions. You have 25 minutes to complete this quiz. You may not use any sources of information, including electronic devices, textbooks, or notes. Leave at least one seat between yourself and other students. Please write clearly. If we cannot read your writing, it will not be graded.

Honor Code. This course operates under the rules of the College of Engineering Honor Code. Your signature endorses the pledge below. After you finish your exam, please sign on the line below:

I have neither given nor received aid on this examination, nor have I concealed any violations of the Honor Code.

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1) **(6 points, -2 for each wrong/blank answer, minimum 0)**
For each of the following mappings indicate what type of function they are (if any). Use the following key:
   i) Not a function
   ii) A function which is neither onto nor one-to-one
   iii) A function which is onto but not one-to-one
   iv) A function which is one-to-one but not onto
   v) A function which is both onto and one-to-one

a) The mapping *f* from Q^+ *to* Q^+ *defined by* $f(x) = 2x$.

   v $f(x) = f(y)$, or in other words, $2x = 2y$, meaning $x = y$

   $f(y/2) = 2y/2 = y$

b) The mapping *f* from N *to* N *defined by* $f(x) = 2x$.

   iv $f(x) = f(y)$, or in other words, $2x = 2y$, meaning $x = y$

   $f(x)$ is never 1 (in larger space of R, it has one solution which is not an integer)

c) The mapping *f* from $[0, \infty)$ *to* $[0, \infty)$ *defined by* $f(x) = |x - 0.5|$.

   iii $f(0.25) = 0.25 = f(0.75)$, but $0.25 \neq 0.75$

   $f(y + 0.5) = |y + 0.5 - 0.5| = |y| = y$

d) The mapping *f* from R *to* $(0, \infty)$ *defined by* $f(n) = 2^n$.

   v $f(x) = f(y)$, or in other words, $2^x = 2^y$, meaning $\log_2 2^x = \log_2 2^y$, or $x = y$

   $f(\log_2(y)) = 2^{\log_2(y)} = y$

2) **(7 points, -2 per wrong circle/no circle, minimum 0)** Circle each of the following which are true propositions. Rationale or 1 counterexample given. There are many other examples

a) $(A \cap B \neq \phi) \rightarrow ((A - B) \subset A)$ true - An element of the intersection will not be in A-B, so a nonempty intersection means A-B is missing at least one element of A

b) $(A - B = A) \rightarrow B \subset A$

false - $A = \{1\}$, $B = \{2\}$. $\{1\} - \{2\} = \{1\}$, but $\{2\}$ is not a subset of $\{1\}$

c) $(A - B = \phi) \rightarrow (A \cap B = B \cap A)$

true - $A \cap B = B \cap A$ always. trivial proof

d) $(A \subseteq B) \rightarrow |A \cup B| \geq 2|A|$

false - $A = B = \{1\}$. $\{1\} \subseteq \{1\}$, but $|\{1\} \cup \{1\}| = 1 < 2 = 2|\{1\}|$

e) $(A \cap B \cap C) \subseteq (A \cup B)$

true - $A \cap B \cap C \subseteq A \cap B \subseteq A \cup B$

f) $\overline{(A - B)} \cap (B - A) = B$

false - $A = B = \{1\}$. $\overline{(\{1\} - \{1\})} \cap (\{1\} - \{1\}) = U \cap \{\} = \{\} \neq \{1\}$

3) **(8 points)** Given the following premises, form a deductive argument which shows that "*t*" must be true. Provide your reasoning. Some of the rules of inference (beyond the ones you are to have memorized) are listed below. You may of course also use logical equivalences.

1. $\neg p \wedge q$ Primise
2. $r \rightarrow p$ Primise
3. $\neg r \rightarrow s$ Primise
4. $s \rightarrow t$ Primise
5. $\neg p$ Simplification, 1
6. $\neg r$ Modus Tollens, 5, 2
7. s Modus Ponens, 6, 3
8. t Modus Ponens, 7, 4

Rule of Inference	*Tautology*	*Name*
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism
p q $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

4) **(9 points)** Prove that if m and n are positive integers and that m*n is even then either m or n (or both) are even.

Assume for contradiction that $\neg(m\ or\ n\ are\ even)$

m is odd and n is odd by De Morgans

m=2a+1, n=2b+1 for a,b integers by definition of odd

mn=(2a+1)(2b+1)=4ab+2a+2b+1=2(2ab+a+b)+1

mn is odd by definition of odd

but mn is even

our assumption is wrong, so m or n are even