Quiz 3

EECS 203

Spring 2016

Name (Print): ____

uniqname (Print): ____

Instructions. You have 25 minutes to complete this quiz. You may not use any sources of information, including electronic devices, textbooks, or notes. Leave at least one seat between yourself and other students. Please write clearly. If we cannot read your writing, it will not be graded.

Honor Code. This course operates under the rules of the College of Engineering Honor Code. Your signature endorses the pledge below. After you finish your exam, please sign on the line below:

I have neither given nor received aid on this examination, nor have I concealed any violations of the Honor Code.

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1) For each of the following, circle *each statement that is true (that could be zero, one, or more for each question).* **[15 points] Each problem is worth 3 points and you only get the points if you circle all of the correct answers and none of the wrong ones.**

a) $X^2(X+\log_2(X))$ is:

$\Theta(X^3)$ **$O(X^2)$** **$\Omega(X^4)$** **$\Theta(X^2 \log_2(X))$** **$O(X^3 \log_2(X))$**

This Is $\Theta(X^3)$, so it is also $O(X^3 \log_2(X))$

b) Consider the following pseudo code:

```
for (i:=1 to n-3)
    for (j:= i to i+3)
      if(A[i]>A[j])
           swap(A[i],A[j]); //Takes Θ(1) time.
```

This algorithm has a run time of

$\Theta(n^2)$ **$O(n^2)$** **$\Theta(n)$** **$\Omega(1)$** **$\Omega(n^2+1)$**

The inner loop always cycles 4 times times, so it is $\Theta(n)$, making it $\Omega(1)$ and $O(n^2)$

c) The function on the right has a runtime that can be characterized as having a runtime of

```
k:=1
while(k<N)
  k=k*2
  for (j:= 1 to N)
    if(A[i]>A[j])
       A[i]=A[i]+1
```

$\Theta(N^2\log(N))$ **$O(N^2)$** **$\Omega(N\log(N))$**
$\Theta(N^2)$ **$O(N)$**

The outer loop cycles $\log(n)$ times, while the inner loop cycles N, giving $\Theta(N\log(N))$, which is $O(N^2)$ and $\Omega(N\log(N))$

d) If A is a countably infinite set and B is an **un**countably infinite set, then $A-B$ could be

Countably infinite **Uncountably infinite** **Finite**

$Z-R=\{\}$ finite. $Z-(0,1)=Z$ countably infinite.

Let f and g be functions from the set of integers or the set of real numbers to the set of real numbers. We say that $f(x)$ is $O(g(x))$ if there are constants C and k such that

$$|f(x)| \leq C|g(x)|$$

whenever $x > k$. [This is read as "$f(x)$ is big-oh of $g(x)$."]

e) If $f(x)=x^2+4x+1$ and $g(x)=x^2+2$, for which of the following k,C value pairs would $|f(x)|\leq C|g(x)|$ for all x>k?

k=2, C=2; **k=3, C=3;** **k=1, C=4;** **k=4, C=1**

k=3, C=3, and k=1, C=4

2) Provide answers for the following **[6 points, 2 points each. Partial credit will be rare.]**

a) Compute (23*22*17*16*21) mod 5. Show your work.

23*22*17*16*21 mod 5 = 3*2*2*1*1 mod 5 = 12 mod 5 = 2

b) Convert $AF1_{16}$ to base 2.

A F 1
1010 1111 0001

c) Convert 23_{10} to base 2.

$23=16+4+2+1=10111_2$

3) Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod 8$.

If n is odd, $n = 2k+1$. $(2k+1)^2 = 4k^2 + 4k + 1 = 4k(k+1) + 1$. Either k or k+1 is odd, so either way, $4k(k+1)$ is a multiple of 8, so $n^2 \equiv 1 (mod\ 8)$