Quiz 5

EECS 203

Spring 2016

Name (Print): ____

uniqname (Print): ____

Instructions. You have 25 minutes to complete this quiz. You may not use any sources of information, including electronic devices, textbooks, or notes. Leave at least one seat between yourself and other students. Please write clearly. If we cannot read your writing, it will not be graded.

Honor Code. This course operates under the rules of the College of Engineering Honor Code. Your signature endorses the pledge below. After you finish your exam, please sign on the line below:

I have neither given nor received aid on this examination, nor have I concealed any violations of the Honor Code.

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The last question may be fairly tricky, we recommend you consider double checking your other answers before starting on it.

1) Circle each pair of events that are independent of each other
[7 points, -3 per wrong answer, minimum 0]

a) Draw two cards from a standard deck of cards one at a time.
   - The first card is the Ace of Spades
   - The second card is a Heart.

   Not Independent - P(Heart)=1/4, but P(Heart|Ace of Spades)=12/51

b) Draw two cards from a standard deck of cards one at a time.
   - The first card is the Ace of Spades
   - The two cards are both of the same rank

   Independent - P(Same)=13/169=1/13, while P(Same|Ace of Spades)=1/13

c) Roll three standard 6-sided dice one at a time.
   - The first two dice are a double (both have the same number)
   - You roll a triple (all three numbers are the same)

   Not independent - P(triple)=6/6^3=1/36, but P(triple|double)=1/6

d) Roll three standard 6-sided dice one at a time.
   - The first two dice are a double (both have the same number)
   - The second die is the same as the third die

   Independent - P(second=third)=6/36=1/6, while
   P(second=third|first=second)=1/6

2) Company X has two spatula factories, one in Japan and one in Mexico. The one in Mexico produces 70% of the company's spatulas and has a defect rate of 1 in 1000 (one spatula in a thousand is defective). The factory in Japan has a defect rate of 1 in 600. If a randomly selected spatula is found to be defective, what is the probability it was produced in Mexico? *You need not supply a number as an answer*—you can simply provide an answer that can trivially be input into a calculator. **[8 points]**

J=made in Japan, M=made in Mexico, D=defective
Given P(M)=0.7, P(J)=1-P(M)=0.3, P(D|M)=0.001, P(D|J)=0.00166, Find P(M|D)

$$P(M|D) = \frac{P(D|M)P(M)}{P(D|M)P(M)+P(D|J)P(J)} = \frac{0.001\cdot 0.7}{0.001\cdot 0.7+0.00166\cdot 0.3} = \frac{0.0007}{0.0007+0.0005} = \frac{7}{12} \approx 0.5833$$

3) Find a closed form version of the following recursion relation: F(x)= 4*F(x-2); F(0)=3; F(1)=2.
[10 points]

$r^x = 4r^{x-2}$, meaning $r^2 = 4$, so $r = \pm 2$

$F(x) = \alpha_1 2^x + \alpha_2(-2)^x$

$F(0) = \alpha_1 2^0 + \alpha_2(-2)^0$, meaning $3 = \alpha_1 + \alpha_2$

$F(1) = \alpha_1 2^1 + \alpha_2(-2)^1$, meaning $2 = 2\alpha_1 - 2\alpha_2$

Adding twice the first equation to the second, we get $8 = 4\alpha_1$, or $\alpha_1 = 2$

From this we can conclude that $\alpha_2 = 3 - \alpha_1 = 1$

Thus, $F(x) = 2^{x+1} + (-2)^x$

4) A computer system considers a string of decimal digits to be a valid code word if and only if the sum of the digits is congruent to 0 mod 9. For instance, 49500, 00000, and 98109 are valid, whereas 29046 and 00008 are not. Let V(n) be the number of valid n-digit code words; find a recurrence for V(n) along with a sufficient number of initial cases. ***You must clearly justify your answer***. (*Hint*: notice that the number of not-valid codes is equal to 10^n-V(n).)
[5 points]

Given an arbitrary string of length n-1, its digits will sum to some value x mod 9. There is only 1 digit in the range of $1 \le y \le 9$ that we can append to the end to make the string of length n have digits totaling a multiple of 9, namely 9-x. This forms a 1-1 correspondence between arbitrary strings of length n-1 and valid code words of length n that end in a non-0 digit. What remains to be found is the number of code words that end in a 0. The sum of the digits is the same as the sum of the digits not including the 0, so we can take any code word of length n-1 and append a 0 to get a code word of length n.

$V(n) = V(n-1) + 10^{n-1}$