EXAM 1—makeup

EECS 203

Spring 2015

Name (Print): ____

uniqname (Print): ____

Instructions. You have 110 minutes to complete this exam. You may have one page of notes (8.5x11.5 two-sided) but may not use any other sources of information, including electronic devices, textbooks, or notes. Leave at least one seat between yourself and other students. Please write clearly. If we cannot read your writing, it will not be graded.

Honor Code. This course operates under the rules of the College of Engineering Honor Code. Your signature endorses the pledge below. After you finish your exam, please sign on the line below:

I have neither given nor received aid on this examination, nor have I concealed any violations of the Honor Code.

____

Problem #	Points
1	/15
2&3	/10
4	/10
5	/15
6	/12
7	/16
8	/10
9	/12
Total	**/100**

1. Logic and sets (15 points)

In this section, each question will have zero or more correct answers. You are to circle each correct answer and leave uncircled each incorrect answer.

[3 points each, -1 per incorrect circle/non-circle, minimum 0 points per problem]

a) Circle each of the following that is a *tautology*.

$(\boldsymbol{a} \rightarrow \boldsymbol{b}) \leftrightarrow (\neg \boldsymbol{b} \rightarrow \mathrm{a})$

$(\boldsymbol{a} \rightarrow \boldsymbol{b}) \rightarrow (\neg \boldsymbol{b} \rightarrow \neg \boldsymbol{a})$

$(\boldsymbol{a} \vee \boldsymbol{b}) \rightarrow \boldsymbol{a}$

$(\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{c}) \rightarrow (\boldsymbol{a} \vee \boldsymbol{c} \vee \boldsymbol{b})$

b) Circle each of the following that is *satisfiable*.

$(\boldsymbol{a} \vee \boldsymbol{b}\ ) \wedge (\boldsymbol{a} \vee \neg \boldsymbol{b}) \wedge (\neg \boldsymbol{a} \vee \boldsymbol{b}) \wedge (\neg \boldsymbol{a} \vee \neg \boldsymbol{b})$

$(\boldsymbol{a} \rightarrow \boldsymbol{b}) \rightarrow (\neg \boldsymbol{b} \rightarrow \neg \boldsymbol{a})$

$(\boldsymbol{a} \vee \boldsymbol{b}) \rightarrow \boldsymbol{a}$

$(\boldsymbol{a} \vee \boldsymbol{b}) \wedge (\neg \boldsymbol{b}) \wedge (\neg \boldsymbol{a})$

c) Circle each of the following which are *tautologies*.

$(\boldsymbol{A} \subset (\boldsymbol{A} \cap \boldsymbol{B}) \rightarrow (|\boldsymbol{B}| = |\boldsymbol{A}|))$

$|\boldsymbol{A} \cup \boldsymbol{B}| \geq |\boldsymbol{A}| + |\boldsymbol{B}|$

$\big((\boldsymbol{A} - \boldsymbol{B}) = \{\boldsymbol{\phi}\}\big) = (|\boldsymbol{A}| = |\boldsymbol{B}|)$

$(\boldsymbol{A} - \boldsymbol{B} = \{\mathbf{1}\}) \rightarrow ((\mathbf{1} \in \boldsymbol{A}) \vee (\mathbf{1} \notin \mathbf{B}))$

$(\boldsymbol{A}\ -\ \boldsymbol{C}\ =\ \boldsymbol{B}\ -\ \boldsymbol{C}) \rightarrow (\boldsymbol{A}\ =\ \boldsymbol{B})$

d) We define the following predicates:

- A(x): x is an adult
- D(x): x hates driving
- M(x): x is Michigander
- S(x): x is a sixty year-old

Consider the statement "Every adult that hates driving is either a Michigander or a sixty year-old." Zero or more of the expressions below are accurate translations of this statement. Circle each of the following which are correct translations, and note that the domain of discourse of x is the set of people.

$\forall x A(x) \wedge D(x) \wedge (M(x) \vee S(x))$

$\forall x (A(x) \wedge D(x)) \rightarrow (S(x) \vee M(x))$

$\forall x ((A(x) \wedge D(x)) \rightarrow M(x)) \vee ((A(x) \wedge D(x)) \rightarrow S(x))$

$\forall x A(x) \rightarrow D(x) \wedge (M(x) \vee S(x))$

e) Circle each of the following statements that are true

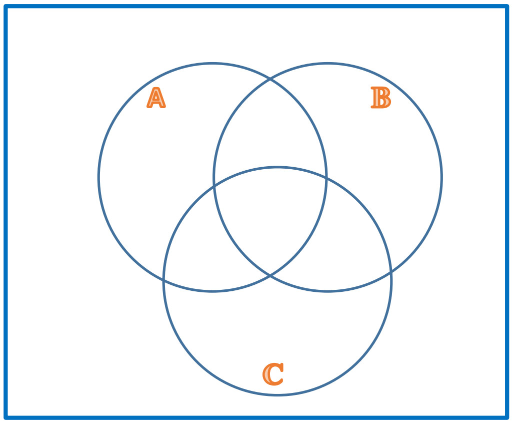
- **The set Z has the same cardinality as N.**
- **The set R has the same cardinality as N.**
- **The set Q has the same cardinality as N.**
- **The set {1, 2, 3} and the set N are both countable.**
- **The set Q+ and R are both uncountable.**

(Here, N denotes the set of natural numbers, Z denotes the set of integers, R denotes the set of real numbers, Q denotes the set of rational numbers, and Q+ denotes the set of positive rational numbers.)

2. More Sets (4 points)

(No partial credit will be given on this problem.)

Shade the Venn diagram below to show $(B \cup A) \cap \left(\overline{C} \cup \overline{B}\right) \cap (B \cup C)$



3. Functions (6 points)

In this section, each question will have zero or more correct answers. You are to circle each correct answer and leave uncircled each incorrect answer.

[3 points each, -1 per incorrect circle/non-circle, minimum 0 points per problem]

a) Say that f is a function from $A \rightarrow B$ where A and B are both subsets of $\aleph$ (the natural numbers). If |B|>|A| then you can conclude that

f is not onto

f is not one-to-one

f is a bijection

f is not a bijection

The cardinality of A is greater than or equal to the cardinality of the range of *f*.

b) Say that f is a function from $A \rightarrow B$ where A and B are both subsets of $\aleph$ (the natural numbers). If $B \subset A$ then you can conclude that

f is not onto

f is not one-to-one

f is a bijection

f is not a bijection

The cardinality of A is greater than or equal to the cardinality of the range of *f*.

4. Growth of functions and infinite sets (10 points)

In this section, each question will have zero or more correct answers. You are to circle each correct answer and leave uncircled each incorrect answer.

Each problem is worth 2 points and you only get the points if you circle all of the correct answers and none of the wrong ones.

a) log(X)+X is:

$\Theta(X)$ **$O(X^2)$** **$\Omega(X)$** **$\Theta(\log(X))$** **$O(\log(X))$**

b) Consider the following pseudo code:

```
for (i:=1 to n)
    x=1;
    while(x<n)
        x=x*2;
```

This algorithm has a run time of

$\Theta(n)$ **$\Theta\ (n^2)$** **$\Omega\ (\log(n))$** **$O(n^3)$** **$O(n/2+1)$**

c) $X^4+12X^2\log(X)+X$ is:

$\Theta(X^4)$ **$O(X^4)$** **$\Omega(X^4)$** **$\Theta(X^3)$** **$\Omega(X^3)$**

d) If A and B are both countably infinite sets, then $A - B$ could be

Countably infinite **Uncountably infinite** **Finite**

e) If A and B are both <u>un</u>countably infinite sets, then $A \cup B$ could be

Countably infinite **Uncountably infinite** **Finite**

5. Number theory questions (15 points)

Provide your answers below and provide work when requested. Partial credit will not be given for incorrect answers (though it might be if you get the right answer without clear work where it is required).

a. Compute 7^{13} mod 10. Show your work. **[3]**

b. Convert 99 to base 16. **[3]**

c. What is gcd(1335,2700)? **[3]**

d. If the product of two integers is $2^4 3^4 5^2 7^2$ and their greatest common divisor is $2^3 3^4 5$, what is their least common multiple? **[3]**

e. Show that 101 is prime. **[3]**

6. Proof by induction (12 points)

Use induction to prove Prove that 3 divides $n^3 + 2n$ whenever n is a positive integer.

Theorem:

Proof:

Base case:

Induction step:

7. Rules of logic (16 points)

Prove the following logical equivalence (in two different ways):

$$(p \rightarrow (q \rightarrow r)) \equiv \neg(p \wedge q \wedge \neg r)$$

(a) Use a truth table **[6]**

(b) Write a formal proof, justifying each line **[10]**

8. Integer proof (10 points)

Let the domain of discourse be the integers. Give a complete proof, with careful justification, for the following statement.

$\forall a\ \forall b\ a^2 - 4b - 2 \neq 0$

9. Other techniques (12 points, 6 each)

a. Find a bijection from $\Re^+$ to $\Re$.

b. Prove that the product of any three consecutive integers is divisible by 6.