EXAM 1

EECS 203

Spring 2016

Name (Print): ____

uniqname (Print): ____

Instructions. You have 110 minutes to complete this exam. You may have one page of notes (8.5x11.5 two-sided) but may not use any other sources of information, including electronic devices, textbooks, or notes. Leave at least one seat between yourself and other students. Please write clearly. If we cannot read your writing, it will not be graded.

Honor Code. This course operates under the rules of the College of Engineering Honor Code. Your signature endorses the pledge below. After you finish your exam, please sign on the line below:

I have neither given nor received aid on this examination, nor have I concealed any violations of the Honor Code.

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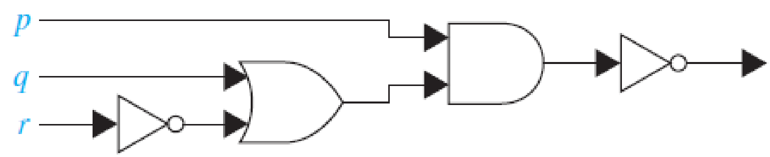
1. Logic (20 points)

In this section, each question will have zero or more correct answers. You are to circle each correct answer and leave uncircled each incorrect answer.

[4 points each, -1.5 per incorrect circle/non-circle, minimum 0 points per problem]

a. Circle each of the following which are equivalent to the circuit below.



$\neg(\boldsymbol{p} \vee (\boldsymbol{q} \wedge \neg \boldsymbol{r}))$

$\neg(\boldsymbol{p} \wedge (\boldsymbol{q} \vee \neg \boldsymbol{r}))$

$(\boldsymbol{p} \vee (\boldsymbol{q} \wedge \neg \boldsymbol{r}))$

$(\neg \boldsymbol{q} \vee \boldsymbol{r}) \wedge \neg \boldsymbol{p}$

$\neg(\neg \boldsymbol{r} \vee \boldsymbol{q}) \vee \neg \boldsymbol{p}$

b. Circle each of the following that are *satisfiable*

$(\boldsymbol{p} \oplus \boldsymbol{q}) \vee (\boldsymbol{p} \vee \boldsymbol{q})$

$(\boldsymbol{p} \wedge \boldsymbol{q}) \rightarrow \boldsymbol{F}$

$\neg(\boldsymbol{p} \vee \boldsymbol{q}) \wedge (\boldsymbol{p})$

$\boldsymbol{T} \rightarrow (\boldsymbol{p} \vee \boldsymbol{q}) \wedge (\neg \boldsymbol{p})$

$\neg \boldsymbol{p} \wedge \neg \boldsymbol{q} \wedge \boldsymbol{p}$

c. Circle each of the following that is a *tautology*

$(\boldsymbol{p} \oplus \boldsymbol{q}) \vee (\boldsymbol{p} \vee \boldsymbol{q})$

$(\neg \boldsymbol{p} \wedge \boldsymbol{p}) \rightarrow (\neg \boldsymbol{q} \wedge \boldsymbol{q})$

$\neg(\boldsymbol{p} \vee \boldsymbol{q}) \wedge (\boldsymbol{p})$

$\boldsymbol{T} \rightarrow ((\boldsymbol{p} \vee \boldsymbol{q}) \vee (\neg \boldsymbol{p}))$

$\neg \boldsymbol{p} \vee \neg \boldsymbol{q} \vee \boldsymbol{p}$

d. We define the following predicates:

   i. F(x): x is Female
   ii. S(x): x is a Student
   iii. K(x,y): x knows y's name

Consider the statement "Jill knows the name of every female student".

Zero or more of the expressions below are accurate translations of this statement. Circle each of the following that are correct.

$$\forall \boldsymbol{x}\big(\boldsymbol{K}(\boldsymbol{Jill},\boldsymbol{x}) \rightarrow \boldsymbol{F}(\boldsymbol{x}) \wedge \boldsymbol{S}(\boldsymbol{x})\big)$$

$$\neg\exists \boldsymbol{x}\big(\boldsymbol{F}(\boldsymbol{x}) \wedge \boldsymbol{S}(\boldsymbol{x}) \wedge \neg\boldsymbol{K}(\boldsymbol{Jill},\boldsymbol{x})\big)$$

$$\forall \boldsymbol{x}\big(\neg\boldsymbol{F}(\boldsymbol{x}) \vee \neg\boldsymbol{S}(\boldsymbol{x}) \vee \boldsymbol{K}(\boldsymbol{Jill},\boldsymbol{x})\big)$$

$$\forall \boldsymbol{x}\left(\big(\boldsymbol{F}(\boldsymbol{x}) \wedge \boldsymbol{S}(\boldsymbol{x})\big) \rightarrow \boldsymbol{K}(\boldsymbol{Jill},\boldsymbol{x})\right)$$

$$\neg\exists \boldsymbol{x}\big(\boldsymbol{F}(\boldsymbol{x}) \wedge \boldsymbol{S}(\boldsymbol{x}) \wedge \boldsymbol{K}(\boldsymbol{Jill},\boldsymbol{x})\big)$$

e. We define the following predicates:

   i. F(x): x is Female
   ii. S(x): x is a Student
   iii. K(x,y): x knows y's name

Consider the statement "Of all pairs of students that know each other's names, at least one in each pair is female".

Zero or more of the expressions below are accurate translations of this statement. Circle each of the following that are correct.

$$\forall \boldsymbol{x}\forall \mathbf{y}\left(\left(\boldsymbol{S}(\boldsymbol{x}) \wedge \boldsymbol{S}(\boldsymbol{y}) \rightarrow \big(\boldsymbol{K}(\boldsymbol{x},\boldsymbol{y}) \wedge \boldsymbol{K}(\boldsymbol{y},\boldsymbol{x})\big)\right) \rightarrow (\boldsymbol{F}(\boldsymbol{x}) \vee \boldsymbol{F}(\boldsymbol{y}))\right)$$

$$\forall \boldsymbol{x}\forall \mathbf{y}\big(\mathbf{S}(\mathbf{x}) \wedge \boldsymbol{S}(\boldsymbol{y}) \wedge \boldsymbol{K}(\boldsymbol{x},\boldsymbol{y}) \wedge \boldsymbol{K}(\boldsymbol{y},\boldsymbol{x})\big) \rightarrow (\boldsymbol{F}(\boldsymbol{x}) \vee \boldsymbol{F}(\boldsymbol{y}))$$

$$\forall \boldsymbol{x}\forall \mathbf{y}\big(\mathbf{S}(\mathbf{x}) \wedge \boldsymbol{S}(\boldsymbol{y}) \wedge \boldsymbol{K}(\boldsymbol{x},\boldsymbol{y}) \wedge \boldsymbol{K}(\boldsymbol{y},\boldsymbol{x})\big) \wedge (\boldsymbol{F}(\boldsymbol{x}) \vee \boldsymbol{F}(\boldsymbol{y}))$$

$$\forall \boldsymbol{y}\exists \boldsymbol{x}\big(\mathbf{S}(\mathbf{x}) \wedge \boldsymbol{S}(\boldsymbol{y}) \wedge \boldsymbol{K}(\boldsymbol{x},\boldsymbol{y}) \wedge \boldsymbol{K}(\boldsymbol{y},\boldsymbol{x})\big) \rightarrow (\boldsymbol{F}(\boldsymbol{x}) \vee \boldsymbol{F}(\boldsymbol{y}))$$

$$\neg\exists \boldsymbol{x}\exists \mathbf{y}\big(\boldsymbol{S}(\boldsymbol{x}) \wedge \boldsymbol{S}(\boldsymbol{y}) \wedge \big(\boldsymbol{K}(\boldsymbol{x},\boldsymbol{y}) \wedge \boldsymbol{K}(\boldsymbol{y},\boldsymbol{x})\big) \wedge \neg(\boldsymbol{F}(\boldsymbol{x}) \vee \boldsymbol{F}(\boldsymbol{y}))\big)$$

2. Sets (6 points)

In this section, each question will have zero or more correct answers. You are to circle each correct answer and leave uncircled each incorrect answer.

[2 points each, -1 per incorrect circle/non-circle, minimum 0 points per problem]

a. Which of the following is the power set of some set?

$$\{\{a\}, \{\}\}$$

$$\{\{\{\}\}, \{\}\}$$

$$\{\{a\}, \{b\}, \{\}\}$$

$$\{\{a\}, \{b\}, \{a, b\}, \{\}\}$$

b. For $i \in Z^+, let\ A_i$ be the set of {0, i, i+1}. So for example, A_2={0,2,3}. Which of the following statements are a tautology?

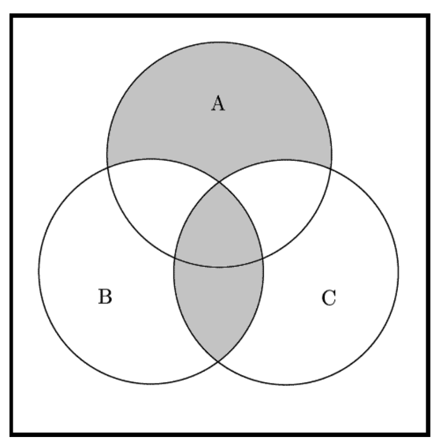
$$\forall x \exists y \left|A_x \cap A_y\right| = 2$$

$$\forall x \forall y \left(A_y - A_x\right) \neq \phi$$

$$\{1, 2\} - A_1 = \{1\}$$

$$\forall x \forall y \left|A_x \cap A_y\right| > 0$$

c. Which of the following correspond to the filled-in part of the following Venn diagram?



$$(A - (B - C)) \cup (B \cap C)$$

$$(A \cap \overline{B} \cap \overline{C}) \cup (B \cap C)$$

$$\overline{B \oplus C} \cup ((A - B) - C)$$

$$(A \cap \overline{B} \cap \overline{C}) \cup (A \cap B \cap C) \cup (\overline{A} \cap B \cap C)$$

3. Cardinality of sets (5 points)

For each of the following, circle *__each statement that is true (that could be zero, one, or more for each question).__* Each problem is worth 1 point and you only get the points if you circle all of the correct answers and none of the wrong ones.

a) If A and B are both **un**countably infinite sets, then A-B could be

Countably infinite **Uncountably infinite** **Finite**

b) If A and B are both **__un__**countably infinite sets, then $A \cup B$ could be

Countably infinite **Uncountably infinite** **Finite**

c) If A is a countably infinite set and B is a countably infinite set, then A-B could be

Countably infinite **Uncountably infinite** **Finite**

d) $(\boldsymbol{Z} \times \boldsymbol{Z} \times \boldsymbol{Z}) - (\boldsymbol{R}^{+} \times \boldsymbol{R}^{+} \times \boldsymbol{R}^{+})$ **is**

Countably infinite **Uncountably infinite** **Finite**

e) $\phi \times Z \times R$ is

Countably infinite **Uncountably infinite** **Finite**

4. Growth of functions (6 points)

For each of the following, circle *each statement that is true (that could be zero, one, or more for each question).* Each problem is worth 2 points and you only get the points if you circle all of the correct answers and none of the wrong ones.

a. Consider the following pseudo code:

```
for (i:=1 to n)
   for (j:= i to n/2)
     if(A[j]>A[i])
          A[i]:=A[i]+1    //Takes Θ(1) time.
for(i:=1 to n)
     doit(i)              //Takes Θ(1) time.
```

This algorithm has a run time that is

$\Theta(n^2)$ **$O(n^2)$** **Θ (n)** **Ω (1)** **Ω (n^2+1)**

b. Using binary search to find a specific element in a sorted list of N values has a run time that is:

$\Theta(\log(N))$ **$O(N^2)$** **$\Omega(N \log(N))$** **$\Theta(N^2)$** **O(N)**

Let f and g be functions from the set of integers or the set of real numbers to the set of real numbers. We say that $f(x)$ is $O(g(x))$ if there are constants C and k such that

$$|f(x)| \leq C|g(x)|$$

whenever $x > k$. [This is read as "$f(x)$ is big-oh of $g(x)$."]

c. If $f(x) = 4x + 5$ and $g(x) = x^2 - 2$, for which of the following k,C value pairs would $|f(x)| \leq C|g(x)|$ for all x>k?

k=3, C=2; **k=2, C=3;** **k=1, C=4;** **k=4, C=1**

5. Functions (8 points)

Answer each of the following questions. Assume all functions are from R to R.

a. What is the inverse of the function, $f(x) = 5x + 1$? **[3]**

$f^{-1}(x) =$ ______________________________

b. If $f(x) = 3x + 4$ and $g(x) = (0.5x + 1)$ what is $(g \circ f)(x)$? **[3]**

$(g \circ f)(x) =$ ______________________________

c. If f(x) is a bijection and g(x) is onto but not one-to-one, $f \circ g(x)$ *might* be (circle all that could apply) **[2]**

one-to-one **onto** **a bijection**

6. Other bases (8 points)

a. How many 0s are at the end of 20! when written in octal? Briefly explain your answer. **[5]**

b. Convert FAD_{16} to base 8. **[3]**

7. Extended Euclidian Algorithm (8 points)

Using the extended Euclidian algorithm, find integer values *a* and *b* such that $60a + 42b = \gcd(60,42)$. You must clearly show your work to get credit. Place your answer where shown.

a=____________ b=____________

8. Finding the sum of a series (6 points)

Find a closed-form solution to the following summation for an arbitrary value of n using any technique. Briefly show/explain your work.

$$\sum_{i=3}^{n} 3i^2 - 3(i-1)^2$$

9. Modular exponentiation. (5 points)

Find the value of $8^{11}\ mod\ 10$. Show your work.

10. Logical arguments (8 points)

Using the premises

i. $p \vee q \rightarrow \neg r$
ii. $(p \vee r) \wedge q$

Provide a formal deductive argument with the conclusion "p". Justify each step.

11. Proof by induction (8 points)

Use induction to prove that $4 | (2n^2 + 6n)$ for all positive integer values of n.

Theorem:

Proof:

Base case:

Induction step:

12. Two last proofs (12 points, 6 each)

a. Prove or disprove that over the domain of integers $\forall x \; [(4|x^2) \lor (4|(x^2-1)]$

b. Find a bijection between Z^+ and the integers divisible by 3.