Final Exam

EECS 203

Spring 2015

Name (Print): ______________________________

uniqname (Print): ______________________________

Instructions. You have 120 minutes to complete this exam. You may have two page of notes (8.5x11.5 two-sided) but may not use any other sources of information, including electronic devices, textbooks, or notes. Leave at least one seat between yourself and other students. Please write clearly. If we cannot read your writing, it will not be graded.

Honor Code. This course operates under the rules of the College of Engineering Honor Code. Your signature endorses the pledge below. After you finish your exam, please sign on the line below:

I have neither given nor received aid on this examination, nor have I concealed any violations of the Honor Code.

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Section#	Points
A	/24
B	/5
C	/11
D	/10
E	/16
F	/10
G	/16
H	/8
Total	**/100**

Section A: Multiple choice (24 points)

In this section, each question will have zero or more correct answers. <u>You are to circle each correct answer and leave uncircled each incorrect answer</u>.

[2 points each, -1 per incorrect circle/non-circle, minimum 0 points per problem]

1) We define the following propositions.

   - p : I will purchase a new iPhone
   - b : The new iPhone can be bent

   Consider the sentence “I will not purchase a new iPhone unless the new iPhone can be bent.”
   Zero or more of the following statements correctly describe the sentence, but which?

   - $b \rightarrow p$
   - $\neg b \rightarrow \neg p$
   - $p \rightarrow b$
   - $b \vee \neg p$

2) For which of the following values of n (if any) makes the following statement true?

   “Whenever 100 balls are distributed between 9 boxes, there must be a box which contains at least n balls.”

   - n = 9
   - n = 10
   - n = 11
   - n = 12

3) Let F(x,y) be the statement that “x can fool y”.

   Circle each logical expression that is equivalent to the statement that “there is someone that no one can fool.”

   - $\exists x \forall y \left(\neg F(x, y)\right)$
   - $\neg \exists y \forall x \left(F(x, y)\right)$
   - $\exists x \forall y \left(F(x, \neg y)\right)$
   - $\exists y \forall x (F(x, y) \rightarrow false)$

4) The statement that $\exists y \forall x\ (x^2 < y)$ is true if the domain of both variables is restricted to the set:

- Z
- Z^+ (positive integers)
- {2, 3, 4}
- (0,1]

5) Circle zero or more of the following which are *tautologies* where A, B and C are subsets of Z.

- $(A \cap B \cap C) \subset (A \cup B)$
- $(A - B) \cap (B - A) = \phi$
- $(B \neq \phi) \rightarrow (|A - B| < |A|)$
- $(A \cap B \neq \phi) \rightarrow ((A - B) \subset A)$

6) Circle all of the following that are TRUE.

- In RSA, the public key, "n" is generally the product of two very large primes.
- The RSA scheme covered in class makes it difficult to convert between the public key and the private key because to do so requires finding the prime factorization of the encryption exponent "e".
- Perhaps the largest difference between a public and private key system is that a private key system relies upon having shared a "key" securely ahead of time, while a public key scheme does not.
- It would be fairly easy to find a private key in RSA if "p" and "q" are found. But we can share "pq" (p times q) safely because finding p and q from the product of p and q is computationally difficult if p and q are large enough.

7) Which of the following are independent of each other (circle zero or more)

- Flip a coin three times
  A: The first coin is a tail
  B: The second coin is heads

- Flip a coin three times.
  A: The first coin is a head
  B: All three coin flips are not all the same (not all heads or all tails)

- Draw two cards from a deck of 52 cards
  A: The first card is a heart
  B: The second card is an ace

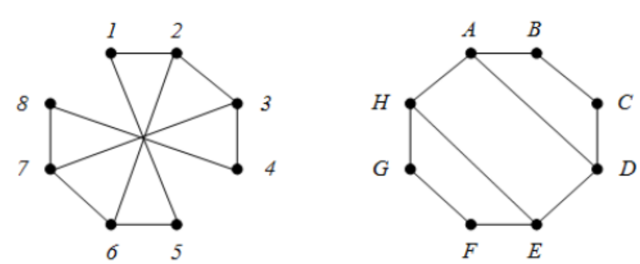
- Roll two dice
  A: The first die is a "6"
  B: The total of the two dice is "7"

8) Consider an application whose runtime can be expressed as $f(n)=4f(n/3)+4n$. Which of the following (if any) would be true of the runtime?

- It is $O(n)$
- It is $O(n^2)$
- It is $O(n \log n)$
- It is $O(n^{\log(n)})$
- It is $O(n!)$

9) Consider the graphs to the right. Circle each of the following (if any) which are true.

- The graphs are isomorphic
- One or both graphs are bipartite
- One or both has an Euler circuit
- Both graphs are planar



10) The equation $3N^2+4N*\log(N)$ is

- $\Theta(N^2\log(N))$
- $O(N^2)$
- $\Omega(N^2\log(N))$
- $\Theta(N^3)$
- $O(N^3)$

11) The function on the right has a runtime that can be characterized as having a runtime of

```
k:=1
while(k<N)
  k=k*2
  for (j:= 1 to N)
    if(A[i]>A[j])
       A[i]=A[i]+1
```

- $\Theta(N^2\log(N))$
- $\Theta(N^2)$
- $O(N^2)$
- $\Omega(N\log(N))$
- $O(N)$

12) How many edges could an undirected graph have if its degree sequence is 1, 2, 3, 3, 4?

- 6
- 7
- 7.5
- 12

Section B: Functions (5 points)

Each part has one correct answer. ***(-2 for each wrong or blank answer, minimum 0)***

For each of the following mappings indicate what type of function they are (if any). Use the following key:

i. Not a function
ii. A function which is neither onto nor one-to-one
iii. A function which is onto but not one-to-one
iv. A function which is one-to-one but not onto
v. A function which is both onto and one-to-one

a) The mapping f from $\mathbb{Z}$ to $\mathbb{Z}$ defined by $f(n) = -n + 1$.

i ii iii iv v

b) The mapping f from $\{1, 2, 3\}$ to $\{1, 3\}$ defined by $f(n) = n$.

i ii iii iv v

c) The mapping f from $\mathbb{Z}$ to $\mathbb{Z}$ defined by $f(n) = 1 - 2n$.

i ii iii iv v

d) The mapping f from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = \frac{1}{x^3+1}$.

i ii iii iv v

e) The mapping f from $\{1, 3\}$ to $\{1, 2, 3\}$ defined by $f(n) = n$.

i ii iii iv v

Section C: Numeric answers (11 points)

Answer the following questions, clearly showing your work. Partial credit for incorrect answers will be rare though clearly showing your work will certainly help. You may leave your answer in a form that could be entered into a calculator (so 2^4 rather than 16 or even something like C(5,2)-C(3,1) is fine).

1) How many trinary strings of length 6 start with 00 and end with a 1? **[3]**

2) Say you have 3 standard (six-sided) dice. What is the probability of getting a total of 17 or lower? **[3]**

3) Say you are getting 9 ice cream sandwiches. Say that there are 3 types: Mint, Chocolate, Resse's and Plain. If there are only 2 Mint ice cream sandwiches and only 3 Plain (and plenty of the other two), how many different ways could you select the ice cream sandwiches? **[5]**

Section D: Probability (10 points)

A given day can be forecast to be rainy, cloudy or clear.

- In Ann Arbor 30% of all days are forecast to be clear, 50% are forecast to be cloudy and 20% are forecast to be rainy.
- On days forecast to be clear, it rains 5% of the time.
- On days forecast to be cloudy it rains 50% of the time.
- On days forecast to be rainy, it rains 80% of the time.

Answer the following questions. *You may leave your answer in a form that could be entered into a calculator (so 2^4 rather than 16 or even something like (0.2*3+0.3)/2 is fine).* <u>Show your work.</u>

1) What is the probability that it rains on any given day? **[3]**

2) What is the probability that if it does rain, the forecast was for a clear day? **[7]**

Section E: Logic proofs (16 points)

1) Show that $\neg(p \lor q) \rightarrow (\neg p \lor \neg q)$ are is a tautology by developing a series of logical equivalences. **[9]**

2) Prove or disprove the following argument using a truth table: if *(p∨r)*, *(¬p∨r)*, and *(p∨q)* are all true then (q ∨ r) is true. *<u>To receive credit, you **must** explain how your truth table proves the result</u>*. **[7]**

p	**q**	**r**					
F	F	F					
F	F	T					
F	T	F					
F	T	T					
T	F	F					
T	F	T					
T	T	F					
T	T	T					

Section F: Proof by induction (10 points)

Use induction to prove the following (you must use induction, any other proof technique will get zero points).

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

Section G: Misc. Problems (16 points)

1) Prove that there exist distinct two powers of 2 that differ by a multiple of 222. That is, $\exists x\ \exists y\ (333|(2^y - 2^x))$ where x and y are positive integers and x≠y. **[8]**

2) Find a recurrence relation for the number of ternary strings of length n that do not contain either 00 or 11.

Section H: Solving recurrence relations (8 points)

Solve the recurrence relation $a_n = -4a_{n-2}$ for $n \geq 2$, $a_0 = 6$, $a_1 = 0$