Quiz 5

EECS 203

Spring 2015

Name (Print): ____

uniqname (Print): ____

Instructions. You have 25 minutes to complete this quiz. You may not use any sources of information, including electronic devices, textbooks, or notes. Leave at least one seat between yourself and other students. Please write clearly. If we cannot read your writing, it will not be graded.

Honor Code. This course operates under the rules of the College of Engineering Honor Code. Your signature endorses the pledge below. After you finish your exam, please sign on the line below:

I have neither given nor received aid on this examination, nor have I concealed any violations of the Honor Code.

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Page #	**Points**
1	/14
2	/16
Total	**/30**

1) Circle each pair of events that are independent of each other
[7 points, -3 per wrong answer, minimum 0]

a) Flip a coin three times.
   - The first coin is a head
   - All three coin flips are the same (all heads or all tails)

b) Flip a coin three times
   - The first coin is a tail
   - The second coin is a head

c) Roll two dice (six-sided, fair, etc.)
   - The first die is a 4
   - The total is 6

d) Roll two dice (six-sided, fair, etc.)
   - The first die is a 6
   - The total is 7.

2) Suppose that 5% of all bicycle racers use steroids, that a bicyclist who uses steroids tests positive for steroids 96% of the time, and that a bicyclist who does not use steroids tests positive for steroids 2% of the time. What is the probability that a randomly selected bicyclist who tests positive for steroids actually uses steroids? *You need not supply a number as an answer*—you can simply provide an answer that can trivially be input into a calculator.
[7 points]

Let P[St] = 0.05 be the probability that a given person uses steroids, P[Po] be the probability that a given person gets a positive test (we are given P[Po | St] = 0.96 and P[Po | not St] = 0.02).

So by Bayes' Theorem we have P[St | Po] = (P[St]P[Po | St])/P[Po] =
(P[St]P[Po | St])/(P[St][P[Po | St] + P[not St]P[Po | not St])=(0.05*0.96)/(0.05*0.96 + 0.95*0.02)

3) Consider the recursion X(n)=2X(n-1)+2 with the initial condition that of X(1)=4. Using induction prove that a closed-form solution to this recurrence is $X(n)=2^n+2^{n+1}-2$. **[11 points]**

Let P(n) be the statement $X(n)=2^n+2^{n+1}-2$.
Base case (P(1)): X(1) = 2^1 + 2^2 – 2): X(1) = 4 = 2^1 + 2^2 – 2 = 2 + 4 – 2 = 4.

Inductive step (P(n) -> P(n+1)) : suppose that $X(n) = 2^n+2^{n+1}-2$ (P(n)).
Then $X(n+1) = 2X(n) + 2 = 2(2^n+2^{n+1}-2) + 2$= 2^(n+1) + 2^(n+2) – 4 + 2 = 2^(n+1) + 2^(n+2) – 2, and thus P(n+1) is true, completing the induction.

4) A computer system considers a string of decimal digits to be a valid code word if and only if it contains an odd number of zero digits. For instance, 12030 and 11111 are not valid, whereas 29046 is. Let V(n) be the number of valid n-digit code words; find a recurrence for V(n) along with a sufficient number of initial cases. (*Hint*: notice that the number of not-valid codes is equal to 10^n-V(n).) **[5 points]**

There are two ways to construct a valid code of length n from a string of n-1 digits:

1) Take a valid code of length n-1; append a number between 1 and 9 (9V(n-1) ways)
2) Take a non-valid code of length n-1; append a 0 (1*(10^(n-1) – V(n-1)) ways)

Total: V(n) = 9V(n-1) + 10^n – V(n-1) = 8V(n-1) + 10^n