Homework #4, EECS 206, Fall 2002. Due Fri. Oct. 11, by 4:30PM

Notes

- Review the HW policies on HW1!
- Reading: “Part 3” lecture notes, Ch. 3 of text, 3.4.5 supplement (on web), Prof. Wakefield’s Fourier series “quick primer” (on web)

Skills and Concepts

- spectra of sums of sinusoids
- spectra of periodic signals (Fourier series)

Problems

1. [20] Text 3.3. (plot to signal, points: a10, b5, c5)

2. [30] Consider the signal $x(t) = 4 + \cos(2\pi 3t) + \sin^3(5\pi t)$.
   
   (a) [10] Express this $x(t)$ as a sum of complex exponential signals. (Use an inverse Euler identity.)
   
   (b) [10] Plot the spectrum (magnitude and phase) of this signal.
   
   (c) [10] Now consider the signal $y(t) = x(2t)$, and plot the magnitude spectrum of $y(t)$.
   
   (d) [0] Describe how this time scaling affected the spectrum. What about a time shift?

4. One of the important topics that we will discuss later in the course is **filtering**, which means processing a signal in a way that amplifies or attenuates various frequency components. The tone controls (bass and treble) in a stereo system are examples of filters: when you “turn up the bass” you are amplifying the low frequency components, and when you “turn down the treble” you are attenuating the high frequency components. This problem is a preview of filters.

You will analyze what happens when a signal \( x(t) \) passes through a special kind of filter called a **moving average** filter. This filter is used frequently in signal analysis to “smooth out” signals. A block diagram for a filter looks like

\[
\text{(Input)} \ x(t) \rightarrow \text{[filter]} \rightarrow \ y(t) \ (\text{Output}).
\]

For this problem, assume that the input signal \( x(t) \) has the spectrum described by the following (amplitude, frequency) pairs \( \{(2 + j2, -8), (4e^{-j\pi/3}, -3), (1, 0), (4e^{j\pi/3}, 3), (2 - j2, 8)\} \), where the frequencies are in Hz.

(a) Plot the magnitude spectrum of \( x(t) \).

(b) Express \( x(t) \) as a sum of sinusoidal signals. Hint: there are three terms, and one of the amplitudes is \( 4\sqrt{2} \).

(c) A \( T \)-second moving average filter is described by the following formula:

\[
y(t) = \frac{1}{T} \int_{0}^{T} x(t + \tau) \, d\tau.
\]

Using this formula with \( T = 1/4 \) s and the formula for \( x(t) \) derived in the previous part, determine \( y(t) \). Your integration should give you a sum of a few sinusoidal signals and you should simplify the result using phasor methods.

(d) Plot the magnitude spectrum of \( y(t) \).

(e) Describe qualitatively how the spectrum of the output signal compares to the spectrum of the input signal.

(f) Repeat the above for a generic sinusoidal signal \( \cos(2\pi f t) \), and make a plot of the output amplitude as a function of the input frequency \( f \). Does this filter amplify or attenuate the high or low frequency components?

5. Problem 2 in Part 3 lecture notes (Fourier series of simple periodic signals)

Part (a) will not be graded. The answer to part (a) is \( \alpha_k = T_0 \cdot \left\{ \begin{array}{ll} 1/2, & k = 0, \\ \frac{1}{2\pi k}, & k \neq 0. \end{array} \right. \)

To do the integral using Matlab’s symbolic toolbox, you could try the following command:

\[
\text{pretty(int('t * exp(-i*2*pi*k*t)', 't', 0, 1))}
\]

6. Problem 4 in Part 3 lecture notes (signal properties from spectra)

7. Problem 6 in Part 3 lecture notes (power of sum of harmonic sinusoids)

Hint: the very hard way to do this would be to use the original power formula in the Part 1 lecture notes. Don’t do it that way!

8. Problem 10 in Part 3 lecture notes (Fourier series of even and odd signals)