Why study spectrum?

- Music synthesis, and other entertainment
- Long distance communications
- Matching a signal to channel, free-space, wire, CD, DVD, etc
- Recognition of speech/audio/video
- Filtering-out noise and disturbances
- Hiding a signal within another signal

Spectral Analysis of Continuous Time Signals

- Start with spectrum of a complex exponential:

\[ x(t) = e^{j\omega_0 t} \]
\[ \omega_0 = 2\pi f_0 \]

Spectrum of Real Sinusoid

- Real sinusoidal signal

\[ X(f) = A \frac{e^{j\omega_0} + e^{-j\omega_0}}{2} = A \frac{\cos(\omega_0 f) + \cos(-\omega_0 f)}{2} \]

Note: Conjugate symmetry

Spectrum of many Sinusoids

- Superposition of sinusoids at different frequencies

\[ X(f) = A_1 \cos(\omega_1 f + \phi_1) + A_2 \cos(\omega_2 f + \phi_2) \]

Fourier Synthesis

Combinations of sinusoids in just the right proportions yields a sawtooth wave [\( \phi_0 = 0 \)]

\[ s(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k f + \phi_k) \]

k-th harmonic frequency:

\[ \omega_k = k \omega_0 \]
If $x(t)$ is smooth and periodic with period $T = 2\pi/\omega_0$, then:

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \phi_k)$$

$$x(t) = x_0 + \sum_{k=1}^{\infty} X_k e^{jk\omega_0 t}$$

Fourier (Spectral) Analysis

- Is it possible to synthesize an arbitrary periodic signal by a sum of sinusoids?
  - Yes! If $x(t)$ is smooth and periodic with period $T = 1/f_0 = 2\pi/\omega_0$.
- Then $x(t)$ can be represented as a sum of sinusoids at frequencies $f_0, 2f_0, 3f_0, \ldots$.
Computing Fourier Coefficients

\[ \alpha_k = \frac{1}{T} \int_{0}^{T} x(t) e^{-j2\pi ft} \, dt \]

x(t) \rightarrow \alpha_k

Analysis - Synthesis

What’s so special about it?

• This “filter bank: structure is ubiquitous in MP3, MPEG-4, JPEG-2000, and other digital standards.
• Now it remains to
  – Explain why Fourier series works
  – Do a couple of examples
  – Discuss some important properties
  – Extend to discrete time (sampled) signals