EECS 206, Fall 2001: HW #11

Due Wednesday, December 5

Problem 1 (parts a-b of the De-reverberation problem below)

Problem 2 (parts c-e of the De-reverberation problem below)

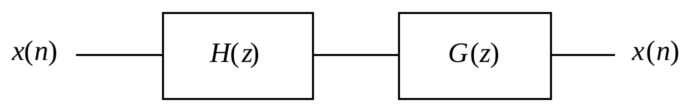
Problem 3 (parts f-g of the De-reverberation problem below)

The De-reverberation Problem. To a first approximation, reverberation is due to receiving sound via reflection paths as well as a direct path. Accordingly, reverberation can be modelled by a finite-impulse response system of the form

$$h(n) = \sum_{k=0}^{K} \beta_k \delta(n - r_k)$$

where each term in the sum represents one path, with r_k representing the reflection tiem for that path and β_k representing the path strength or gain. Without loss of generality, the *direct path* has zero delay and unity gain (i.e., $\beta_0 = 1$, $r_0 = 0$). In general, the gains are less than one for the reflected paths (i.e., $0 < \beta_k < 1, r_k > 0, \forall k > 0$).

a. For the general form of the reverberation model above, find the system function $G(z)$ that eliminates the reverberation. That is, find the $G(z)$ such that



b. Suppose $K = 1$. Consider the special case of one reflection, where $r_1 = 4$ and $\beta_1 = 0.5$. Write a closed-form expression for the frequency response of H, G, and the cascaded system GH. Using Matlab, graph the magnitude of each of these frequency responses (in dB) in the same figure. Clearly label your axes and each of the frequency response curves.

*****For parts c-g, let $K = 1$, $r_1 = 4$ and $\beta_1 = 0.5$.*****

c. De-reverberators are "easy" to design, given exact knowledge about the times of reflection ($\{r_k\}$) and their gains ($\{\beta_k\}$). If these parameters aren't known exactly, problems can develop. Suppose you know β_1 to within a 20% margin of error for the purposes of designing the de-reverberator. Denote your estimate of β_1 by $\hat{\beta}_1$. By examining the frequency response of the cascaded system GH, show that the poorest de-reverberation occurs at the extremes of the error, that is, for $\hat{\beta}_1 = 0.4$ and $\hat{\beta}_1 = 0.6$. As in part b), plot the magnitude of the frequency responses of H, G and GH for these two extremes on two separate figures. Clearly label your axes and each of the frequency response curves. For each figure, what is the maximum deviation in the magnitude spectrum of GH when compared with the ideal case for β_1 known exactly?

d. Consider the effects of not knowing the exact reflection time. Suppose you are off by a *single* sample, that is $\hat{r}_1 = 3$ or $\hat{r}_1 = 5$. Plot the magnitude of the frequency responses of H, G and GH for these two extremes on two separate figures. For each figure, what is the maximum deviation in the magnitude spectrum of GH when compared with the ideal case for β_1 known exactly?
e. Explain your results in parts b, c, and d by in terms of the locations of the poles and zeros of H, G, and GH. Use the Matlab command *zplane* to support your arguments.
f. In designing such systems, we talk about the sensitivity of the design to errors in our knowledge. What percentage margin of error in gain β_1 would result in the same maximum deviation in the magnitude spectrum of GH from the ideal case for the case of $\hat{r}_1 = 3$. Develop a percentage error measure for reflection delay and indicate for which of the two parameters, gain or delay, the design is more sensitive.
g. What happens to your design in part b) if the laws of physics are violated and reflective surfaces actually put more energy *into* the signal? That is, what happens if $\beta_1 > 1$?

4. Which of the following pairs of signals are uncorrelated/orthogonal? In each case where both signals are periodic, correlate over the least common multiple of their fundamental periods.

(a) $x(t) = (-1)^{\text{floor}(t/3)}$, $y(t) = (-1)^{\text{floor}(2t/3)}$

(b) $x(t) = \cos(2\pi 100t)$, $y(t) = 3\sin(2\pi 200t)$.

Hint: $\cos\alpha \cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta)$.

(c) $x(t) = t - 1$, $y(t) = -t + 1$. Correlate over the time interval $[0,1]$.

5. Let $y(t) = x(t) + n(t)$, where $x(t) = 3t$, $0 \le t \le 1$, and $x(t) = 0$ for all other t, where $n(t)$ has zero mean, and where $x(t)$ and $n(t)$ are orthogonal/uncorrelated. Find $c(x,y)$, the correlation between x and y over the time interval $[-1,2]$. (Note that $x(t)$ does not have zero mean.)

6. Let $u[n] = \delta[n] + \delta[n-1] + \delta[n-2]$.

(a) Find the impulse response $h[n]$ of the FIR filter that implements sliding correlation of an arbitrary signal $x[n]$ with $u[n]$, where $u[n]$ is the "slider". (Note that $u[n]$ does not have zero mean.)

(b) Find and sketch the output of the filter found in part (a) when the input is $u[n]$.

7. Let $u[n]$ be an arbitrary signal such that $u[n] =$ for $n<0$ and $n>M$. Let $h[n] = u[M-n]$ be the impulse response of the filter that implements sliding correlation of other signals with $u[n]$. Let $y[n]$ be the output of this filter when the input is $u[n]$.

(a) Show that $y[M] = E(u)$.

(b) Show that $y[M+n] = y[M-n]$ for every n.

(c) Show that $|y[n]| \le E(u)$ for every n .

Note: Together, your work in (a), (b) and (c) shows that the output signal $y[n]$ is symmetric about its midpoint, where it reaches a peak value that is the energy of $u[n]$.