Homework Set 12

Relevant lectures: 12/03, 12/05 and early lectures on the DFT and sampling Relevant notes: The posted lecture notes for the 12/03 and 12/05 lecture .

- 1. 9.1, p. 376 (When plotting the spectrum, label each spectral line with its complex amplitude and make the height of the line proportional to ithe magnitude of the complex amplitude.) (Hint for Part c: You might try an approach motivated by Part b.)
- 2. 9.2, p. 376 (Assume each signal is periodic with period 10.)
- 3. 9.3, p. 376 (Assume each signal is periodic with period 10.)
- 4. Working directly from the DFT coefficients, find the power of each of the periodic signals whose DFT's are given Problem 3, Parts a, b, c.
- 5. An FIR filter has coefficients  $\{b_k\} = \{3,1\}$ . Find its output when the input is the signal periodic signal whose 10 point DFT is given in Problem 3a. Repeat for parts b, c, d. In each case, simplify the expression for the input as much as possible.
- 6. Let x(t) be a periodic continuous-time signal with fundamental period  $T_0 = 0.001$ , with fundamental period  $f_0 = 1/T_0 = 1000$  hz, and with highest frequency component at frequency 9,600 hz. We wish to sample this signal at some sampling rate  $f_s = 1/T_s$  and then apply the DFT to the discrete-time signal x[n] = x(nTs) in order to determine the spectrum of x(t).

(a) How large must the sampling rate be in order for x(t) to be perfectly reconstructable from the discrete-time signal x[n]?

(b) Show that if  $f_s$  is an integer multiple of  $f_o$ , i.e.  $f_s = M f_o$ , then x[n] is periodic with period M.

(c) Find the smallest sampling rate  $f_s$  such that x(t) can be perfectly reconstructed from its samples AND x[n] is periodic.

(d) Let the sampling rate be as chosen in Part (c), and let N be a multiple of the M that corresponds to your answer in Part (c). Applying an N-point DFT to the x[n]'s yields coefficients X[0],...,X[N-1]. What are the frequencies in hz of the complex exponential components of x(t) to which these coefficients correspond?

(e) Suppose we wish the complex exponential components of x(t) to be separated by 500 hz. What should N be?

## Homework submission policies:

Write neatly and legibly. The graders will not grade papers that are illegible or difficult to read. Submit the problems in the assigned order.

Clearly write your name, lecture session number, and lab session number at the top of your paper. Staple your paper in the upper left corner.

Hand in your homework just before or just after the lecture, or place it in the box outside Room 4230D EECS before 5 PM.

See the collaboration policy described on the first day handout and on the website.