EECS 206, Fall 2001: HW #6[1]

Due Wednesday, October 24

Relevant reading materials: Chapter 5, DFT lecture notes

Relevant lectures: 10/8, 10/10, 10/12

Problems 1,2,3: from Chapter 5, numbers 3, 9a, 11

Problem 4

Suppose that a sequence x[n] is simultaneously the input to two FIR filters, whose impulse responses are h1[n] and h2[n] and whose orders are M1 and M2, respectively. Let y[n] be the sum of the outputs of the two filters.
(a) Draw a block diagram showing x[n], the two filters, and y[n],
(b) Show that y[n] may be viewed as the output of a single FIR filter with input x[n].
(c) Find its impulse response and order in terms of what we are given.
Note: In class and in Section 5.7 of the text we have considered the 'cascade' connection of two systems. This problem is concerned with the 'parallel' connection of two systems.

Problem 5

Let $x[n] = \cos(2\pi k_0 n / N)$ where $k_0 \in [0, N-1]$. Using the general formula for the Discrete Fourier Transform, e.g.,

$$X(k) = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N} \qquad k = 0, \ldots, N-1$$

compute the values of $X(k)$ for all k. Hint: expand the cos function using Euler's identities and use the results established in lecture for the summation of the geometric series.

Problem 6

(a) For $x[n]$ in Problem 5, let $N = 12$ and $k_0 = 3$. Verify your general results in Problem 5 by implementing the DFT in Matlab as a pair of nested for loops. Provide a copy of your code. In addition, plot the magnitude of $X(k)$ and the phase of $X(k)$ using the stem plotting routine.
(b) Explain, briefly, how your results confirm the theoretical result you obtained in Problem 5.
(c) How many multiplies did you require to evaluate the DFT? For general N, how many multiplies are required?
(d) - see next page...

1.**Homework submission policies**: Write neatly and legibly. The graders will not grade papers that are illegible or difficult to read. Submit the problems in the assigned order. Clearly write your name, lecture session number, and lab session number at the top of your paper. Staple your paper in the upper left corner. Hand in your homework just before or just after lecture, or place it in the box outside Room 4230D EECS before 5 PM. See the collaboration policy described on the first day handout and on the website.

(d) Repeat your analysis in part a by letting $N = 12$ and $k_0 = 3.5$. Plot the magnitude of $X(k)$ and the phase of $X(k)$ using the stem plotting routine. How many non-zero frequency components do you observe? Why should there be so many non-zero frequency components when $x[n]$ is a cosine function? Is there a choice of N that would result in a spectrum that contains two non-zero frequency components? [Hint: What is the relationship between N and the period of the signal?].