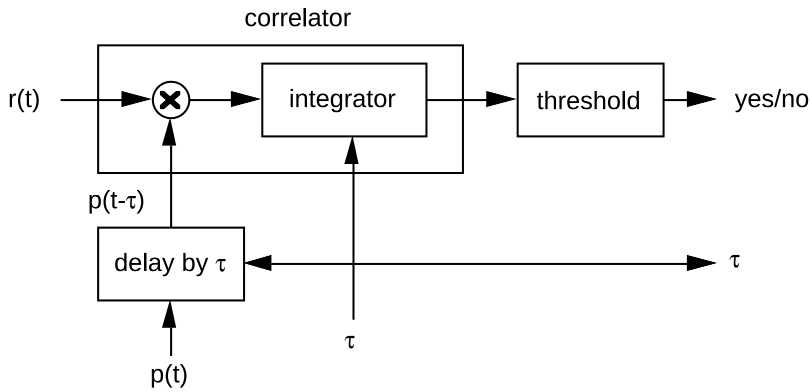
I. Examples of Correlation Based Detection/Classification Systems

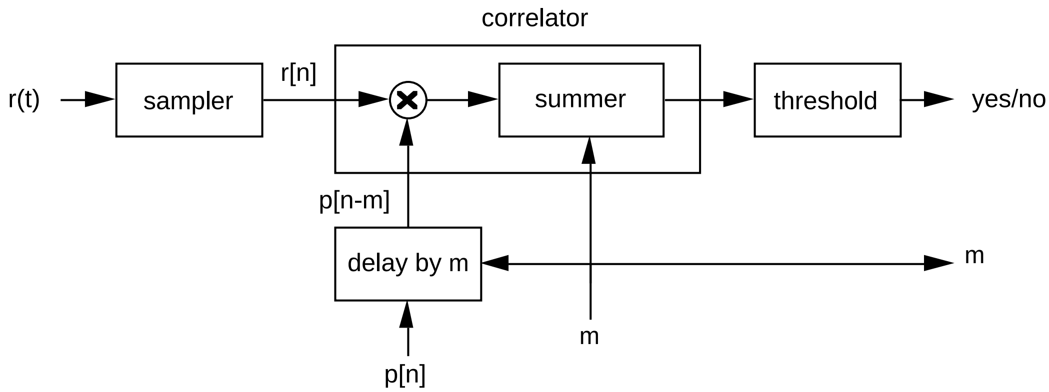
1. Radar/sonar pulse etection

Task: A pulse p(t) is transmitted. An antenna/receiver listens for a reflected pulse. Given the signal r(t) received from the antenna over the time interval $0 \le t \le T$, the detectormust determine if r(t) contains a known radar/sonar pulse p(t), and if so, when. That is, it must find a time τ such that $r(t) = a\, p(t-\tau) + n(t)$, where $a << 1$ represents the attenuation encountered by the pulse, τ represents the time delay encountered by the pulse, and n(t) represents the noise that was also received, or decide that there is no pulse in r(t) in the time interval $0 \le t \le T$.

A continuous-time radar/sonar pulse detector:



A discrete-time radar/sonar pulse detector:



The "coherent" detection method based on correlating, shown above, is more accurate than "noncoherent" ones based simply on signal strength measurements (e.g. envelope detection, as in the telephone dialer decoder of Lab 6).

2. Coherent digital communication signal detection

Task: Given a received signal $r(t)$ determine the sequence of transmitted signal pulses.

Example -- a binary communication system: To transmit a sequence of bits $b[0], b[1], b[2], \ldots$, where each $b_i = \pm 1$, the transmitter sends[2]

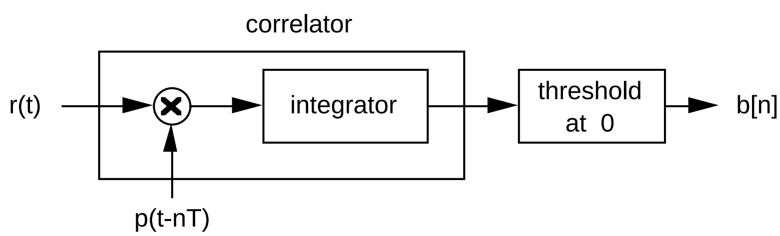
$$s(t) = b[0]\, p(t) + b[1]\, p(t-T) + b[2]\, p(t-2T) + \ldots$$

where $p(t)$ is the basic transmitter pulse. Usually, $p(t) = 0$, when t is not in the interval $0 \le t \le T$. From the antenna, the receiver is given

$$r(t) = a\, s(t) + n(t)$$

where $a << 1$ represents the attenuation encountered by the signal, and $n(t)$ is the noise received by the antenna. The detector must make decisions about the $b[n]$'s.

A correlating (coherent) detector[3]



"Coherent" detection methods based on correlating, such as that shown above, are more reliable (make fewer decision errors) than "noncoherent" ones based simply on signal strength measurements (e.g. envelope detection, as in the telephone dialer decoder of Lab 6).

3. Pattern recognition

Task: Given a signal $r(t)$ determine if it contains a "pattern signal" $p(t)$, and if so where. That is, determine if $r(t) = a\, s(t-\tau) + n(t)$, where a is an amplitude constant and $n(t)$ is noise.

This is essentially the same problem as the radar/sonar detection problem, except that t might represent position rather than time. Indeed, in many cases, the signals are two-dimensional, e.g. images.

Specific application: finger print recognition, retinal eye scan recognition, dollar bill recognizer, automatic part recognition, heart defect detection in EKG, ...

4. Watermarking

Add a faint imperceptible pattern to an image identifying its origin. Use correlation detection to check image origination.

Add a faint imperceptible pattern to a music recording.

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[2]Actually, in many systems the transmitter sends $s(t)\cos(2\pi f_c t)$ which has the effect of translating the signal spectrum into a band of frequencies around some designated frequency f_c. Correspondingly, the receiver has a first stage, not discussed here, that removes the effect of this sinusoidal modulation.

[3]A discrete-time version would have a sampler and would sum rather than integrate.

Sliding-Correlation Can be Implemented as an LTI Filter

Suppose we wish to correlate signal $x[n]$ with time delayed versions of $p[n]$, where $p[n] = 0$ for $n < 0$ and $n > M$. Let $y[n_o]$ be the result of correlating $x[n]$ with $p[n-(n_o-M)]$. The situation is shown below:

$$\begin{array}{cccccccccc} x[1] & x[2] & \ldots & x[n_o-M-1] & x[n_o-M] & x[n_o-M+1] & \ldots & x[n_o-1] & x[n_o] & x[n_o+1] \ldots \\ & & & & p[0] & p[1] & \ldots & p[M-1] & p[M] & \end{array}$$

Then

$$y[n_o] = \sum_{n=0}^{M} x[n_o-n]\, p[M-n] = \sum_{n=0}^{M} x[n_o-n]\, h[n] \qquad (**)$$

where h is just p flipped in time:

$$h[n] = p[M-n].$$

From (**) we see that correlation can be done by FIR filtering with a filter with impulse response $h[n] = p[M-n]$.

We have learned an additional goal of filtering, namely, sliding-correlation. The previously introduced goals of filtering were: smoothing, sharpening, frequency blocking/shaping (low pass filter, high-pass filter, bandpass filter). These goals have mainly to do with the magnitude of the frequency response of the filter. The new goal, sliding-correlation, is something that requires more than just carefully designing the magnitude frequency response. It is essentially a time-domain goal.