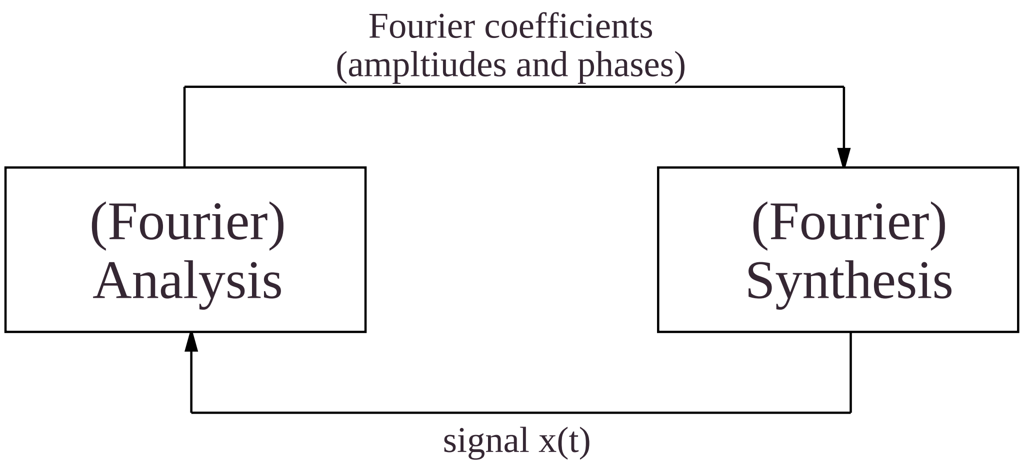
Lab #3 Notes

The focus of Laboratory #3 is the concept of *spectrum.* Supposing we consider a signal that is made up of a sum of sinusoids (and we'll see that *any* signal can be thought of as a sum of sinusoids!), the spectrum of the signal refers to the frequencies, amplitudes, and phases of the sinusoids that make it up. If we know those parameters, we can *synthesize* the signal by simply adding up the appropriate sinusoids. We'll also see that there is another possible step. Given the signal, we can *analyze* the signal to determine the amplitudes, frequencies, and phases of its constituent sinusoids. These two processes are complementary, and they form an *analysis-synthesis loop.* Thus if we analyze a signal and synthesize the result, we should get something identical to the original signal. Similarly, if we have parameters for our sinusoids, we can synthesize a signal and then analyze the result to obtain the original parameters.



Synthesis

Synthesis is the easy part. In order to perform signal synthesis, we simply have to add up sinusoids with the appropriate sinusoids. The mathematical formula for this is

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k) \tag{1}$$

Notice the inclusion of the constant DC term, A_0. In this lab, you'll be writing a function to perform synthesis based on this equation.

A useful special case of this synthesis equation occurs when the frequencies f_k are *harmonically related.* This means that every frequency is an integer multiple of some *fundamental frequency*, f_0. Sometimes we will call sinusoids that are harmonically related *harmonics* of *harmonic partials.* With harmonically related sinusoids, the resulting synthesis signal will be periodic. We can write the equation for this special case (known as *Fourier synthesis*) as

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \phi_k) \tag{2}$$

Analysis

If a signal $x(t)$ is periodic with period T, we know that it can be written as a sum of harmonically related sinusoids with a fundamental frequency $f_0 = \frac{1}{T}$. Through a process called *Fourier analysis* we can extract the *complex amplitudes* of these constituent sinusoids.

These complex amplitudes, X_k are often referred to as *Fourier coefficients.* The formula for Fourier analysis is

$$X_k = \frac{2}{T}\int_0^T x(t)e^{-j2\pi kt/T_0}dt \tag{3}$$

The DC value of the signal (A_0 above) is calculated as

$$X_0 = A_0 = \frac{1}{T}\int_0^T x(t)dt \tag{4}$$

The amplitudes and phases of our original sum-of-cosines representation in equation 2 can be calculated from the Fourier coefficients as

$$A_k = |X_k| \tag{5}$$

$$\phi_k = \arg X_k \tag{6}$$

You won't need to worry about writing MATLAB code to perform Fourier analysis in this course; instead, a function is provided that will perform Fourier analysis for you.

Notice the form of the Fourier analysis equation. In Lab #2, we explored the concept of *correlation*, which compares two signals and returns a number that indicates their similarity. This is exactly the what the Fourier analysis equation does for us: it correlates our signal with a complex exponential. The resulting complex number tells us how "similar" a signal is to a complex exponential of a given frequency. That is, this number (the Fourier coefficient) indicates how strongly a particular frequency is present in the signal and what it's phase shift is.

Visualizing Fourier Coefficients: Using Decibels

If we examine the magnitudes of the Fourier coefficients of a given signal, we often find that some coefficients have much greater magnitudes than others. For instance, we may find that the magnitudes of some coefficients are only $1/100^{th}$ the magnitudes of the largest coefficients. If we plot these coefficients, we will be hard pressed to see and compare smaller coefficients despite the fact that they are usually very important to the signal. So that we can see and compare these coefficients, we often plot them in *decibels* (or *dB*).

To convert a number, x_{la}, from *linear amplitude* (what we usually use) into decibels, we use the following formula:

$$x_{dB} = 20\log_{10} x_{la} \tag{7}$$

Note that if we multiply x_{la} by a constant c, this is equivalent to adding $20\log c$ to x_{dB}. Also, the decibel transformation is only valid for values of x_{la} that are not negative. Often, decibels are used with ratios; in this case, a ratio of one is equivalent to 0 dB. This becomes useful because the reciprocal relationship (often necessary when dealing with ratios) is mapped to simple negation. For instance, the number 1/2 is (roughly) -3 dB while the number 2 is 3 dB.

Sometimes you will see the following form of the decibel transformation:

$$x_{dB} = 10\log_{10} x_{la} \tag{8}$$

We use this formula when we are dealing with "power" measurements, which are the square of our usual amplitudes. Thus, if we have the magnitude squared of our Fourier coefficients, $|X_k|^2 = X_k X_k^*$ (a fairly common form), we would use the equation $x_{dB} = 10\log_{10} x_{la}$.