Notes

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• Reading: Text Ch. 7. NOTE: All problem numbers are the same in DSP 1st and SIG. PROC. 1st
• Relevant practice problems on the DSP CDROM: 7.3, 7.4, 7.18, 7.21, 7.25, 7.32, 7.35, 7.46, 7.49

Skill Problems

1. [0] Text 7.1. Concept(s): \textit{(z-transform of finite sequences)}
   \begin{align*}
   \text{Hint: } X_4(z) &= 2 - 3z^{-1} + 4z^{-3}
   \end{align*}

2. [0] Text 7.3. Concept(s): \textit{(diffeq and response from system function)}
   \begin{align*}
   \text{Hint: } h[n] &= \delta[n] + 5\delta[n-1] - 3\delta[n-2] + 2.5\delta[n-3] + 4\delta[n-8]
   \end{align*}

3. [30] Text 7.8. Concept(s): \textit{(diffeq to } h[n], H(z), \text{zplane, } \mathcal{H}(\omega))

4. [15] Text 7.9ace. Concept(s): \textit{(Cascade of two } H(z) \text{’s)}

5. [15] Concept(s): \textit{system function and frequency response}
   A filter has input-output relation
   \begin{equation*}
   y[n] = x[n-1] + \sqrt{2}x[n-3] + x[n-5].
   \end{equation*}

   (a) [5] By taking the Z-transform of both sides of this equation to obtain \(Y(z) = H(z)X(z)\), identify the system function \(H(z)\).

   (b) [5] From the system function of part (a), show that there is a frequency \(\omega_0\) for which the output signal is zero when \(x[n] = A \cos(\omega_0n + \phi)\) for any \(A, \phi\). What is this frequency?

   (c) [5] Determine the output signal \(y[n]\) when the input is the above sinusoid with \(A = 10, \omega_0 = \pi/5\) and \(\phi = 0\).
6. [25] Concept(s): suddenly applied signals

The input to an LTI system is the following discrete-time signal, which is periodic for $n \geq 0$.

```
-3 -2 -1 1 2 3 4 ...
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The frequency response of the system is given by

$$H(\omega) = \frac{1}{4} \left( 1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} \right).$$

(a) [20] Determine a formula for the output signal $y[n]$ without using braces.
(b) [5] Sketch the output signal $y[n]$, and identify which part is the transient response.

7. [30] Concept(s): filtering of sampled continuous-time signals

The above sawtooth signal $x(t)$ is the input to the following connected systems:

```
<table>
<thead>
<tr>
<th>x(t)</th>
<th>Ideal anti-alias filter</th>
<th>x_a(t)</th>
<th>Ideal C-D</th>
<th>x[n]</th>
<th>LTI</th>
<th>y[n]</th>
<th>Ideal D-C</th>
<th>y(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>f_s = 1500Hz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

(a) [0] Sketch the spectrum of $x(t)$.
(b) [5] Sketch the spectrum of $x_a(t)$.
(c) [10] Sketch the spectrum of $x[n]$.
(d) [10] Sketch the spectrum of $y[n]$, assuming that $h[n] = \frac{1}{2} (\delta[n] + \delta[n-1])$.
(e) [5] Determine $y(t)$.

Optional challenge. Design a filter (specify $h[n]$) that would remove all of the harmonics except the fundamental frequency for the above sampled sawtooth signal.

8. [10] Concept(s): response to mixed periodic / aperiodic input given $H(z)$

Consider the following cascade of LTI systems

```
x[n] \rightarrow H_1(z) = 1 + z^{-2} \rightarrow H_2(z) = 1 - z^{-2} \rightarrow y[n].
```

The input to this cascade system is the signal

$$x[n] = 20 - 7\delta[n - 5] + 20 \cos\left(\frac{\pi}{4}n\right).$$

Determine the output signal $y[n]$. Given an equation (no braces!) valid for all $n$.

Hint. Do not use convolution. Use more than one method and linearity.