Topics: Systems and Filters

1. Concept of Systems
2. Filters
3. Properties of filters
   (a) Linearity
   (b) Time-invariance
   (c) Causality
   (d) Stability

1 Signals and Systems

• Signals and Systems: Lectures mostly about signals so far
• We now study certain kind of systems called filters, in particular linear time-invariant (LTI) filters.
• Lab 6 experiments with filters, mostly LTI, but also a little with nonlinear filters—median filters.
• We concentrate on discrete-time systems and filters.
  (a) There are also continuous-time systems and filters.
  (b) But we can also do continuous-time filtering by sampling, discrete-time filtering and reconstruction.
  (c) EECS 306 deals with continuous-time systems and filters.

2 Systems

(1) What is a system?
   A discrete-time system is an object that
   (a) takes a discrete-time input signal $x[n]$
   (b) and produces a discrete-time output signal (response) $y[n]$ determined by the input signal.

(2) Block diagram

   $x[n] \rightarrow \text{system} \rightarrow y[n]$

(3) Notations
   (i) $x[n] \mapsto y[n]$ means that $x[n]$ is the input and $y(t)$ is the response of the system to $x[n]$. 
(ii) \( y[n] = \mathcal{T}\{x[n]\} \)

\( \mathcal{T}\{\cdot\} \) is an operator that maps a signal into a signal.

(4) Examples of systems

\[
\begin{align*}
y[n] &= x[n - 1] + x[n] \\
y[n] &= x[n] + 3 \\
y[n] &= x^3[n - 1] + 2x[n] \\
y[n] &= x[-n] \\
y[n] &= x[n] \cos(\hat{\omega}n + 0.1\pi)
\end{align*}
\]

\( \begin{array}{c}
x[n] \\
\downarrow \\
y[n] = x[n - 1] + x[n]
\end{array} \)

(5) (Deterministic and nondeterministic)

(i) Our systems are \textit{deterministic}, meaning that the output signals is entirely determined by the input signal.

(ii) An example of a \textit{nondeterministic} system

\( \begin{array}{c}
x[n] \\
\downarrow \\
v[n] \\
\downarrow \\
y[n] \end{array} \)

\( \begin{array}{c}
x[n] \\
\downarrow \\
\times \\
\downarrow \\
\cos(\hat{\omega}n + 0.1\pi) \\
\downarrow \\
y[n] \end{array} \)

where \( v[n] \) is a noise that is not known in advance and not repeatable, e.g., random.

(6) We focus on a special kind of systems called \textit{linear time-invariant} filters.
3 Linear Time-Invariant Filters

(1) Linear time-invariant (LTI) filters are very common.

(2) Examples of LTI filters

(a) (Delay)

\[ y[n] = x[n - 1] \]

(b) (Running/Moving/Sliding Average) smoothing

\[ y[n] = \frac{1}{3}(x[n - 2] + x[n - 1] + x[n]) \]

(c) (Running Sum) smoothes and amplifies

\[ y[n] = x[n - 2] + x[n - 1] + x[n] \]

(d) (Running Average with look ahead)

\[ y[n] = \frac{1}{3}(x[n - 1] + x[n] + x[n + 1]) \]
(i) Running average with look ahead
(ii) Cannot do this in real time
(iii) Can do this on recorded data
(iv) (Causality) Filters that operate in real time are called causal.

The output value at time $n$ depends only on the input values at time $n$ and before, i.e., does not depend on the input values of the future time $x[n+1], x[n+2], \ldots$.

(e) (Weighted running average)

$$ y[n] = \frac{1}{4} x[n-1] + \frac{1}{4} x[n] + \frac{1}{2} x[n+1] $$

Different choices of weights are for different purposes.

| Filters given above have finitely many terms involving the input signal $x[n]$. |
| They are called finite-impulse-response (FIR) filters. |

(f) (IIR Filters) The following are not FIR, by infinite-impulse-response (IIR) filters.

(i)

$$ y[n] = \frac{1}{2} x[n] + \frac{1}{4} x[n-1] + \frac{1}{8} x[n-2] + \cdots. $$

Note the infinite number of terms involving $x[n]$.

(ii)

$$ y[n] = \frac{1}{2} y[n-1] + x[n]. $$

This turns out to be the same as (i).
General Form of LTI Filters:

\[ y[n] = \sum_{k=-M_1}^{M_2} b_k x[n-k], \]

where \( M_1 \) and \( M_2 \) are integers

\[ 0 \leq M_1, M_2 \leq \infty. \]

Classification:

1. (FIR or IIR)
   (i) FIR: \( M_1, M_2 < \infty \).
   (ii) IIR: \( M_1 \) or \( M_2 = \infty \).

2. (Causal or noncausal)
   (i) Causal: \( M_1 = 0 \).
   (ii) Noncausal: \( M_1 > 0 \).