- First-order IIR filter
- Second-order IIR filter

1 First-Order IIR Filter

(a) **Difference equation:** a_1 and b_0 real

$$y[n] = a_1 y[n-1] + b_0 x[n].$$

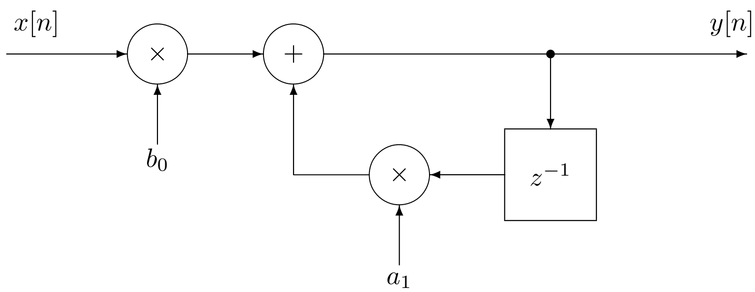
(b) **System function:**

$$H(z) = \frac{b_0}{1 - a_1 z^{-1}} = \frac{b_0 z}{z - a_1}.$$

(c) **Impulse response:**

$$h[n] = b_0 a_1^n u[n].$$

(d) **Implementation:**



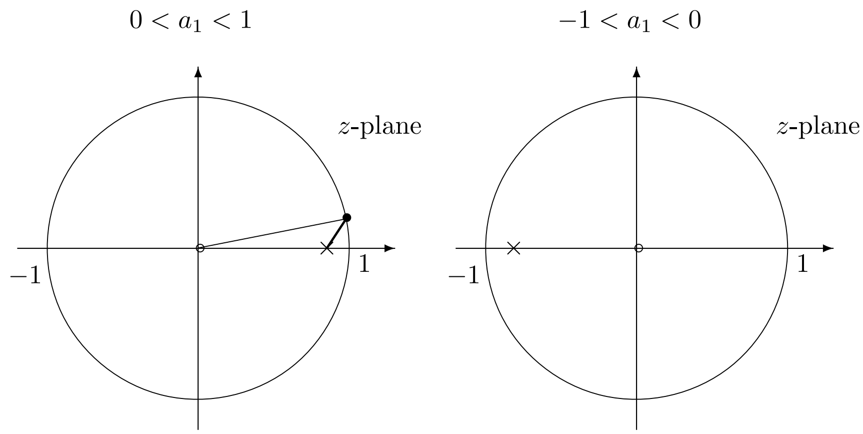
(e) **Stability requirement:** $|a_1| < 1$

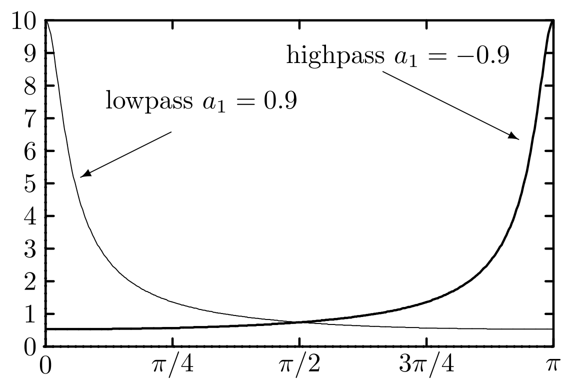
(f) **Frequency response function:**

$$\begin{aligned}\mathcal{H}(\hat{\omega}) &= H(z)\Big|_{z=e^{j\hat{\omega}}}.\\ |\mathcal{H}(\hat{\omega})|^2 &= \mathcal{H}(\hat{\omega})\mathcal{H}^*(\hat{\omega})\\ &= \frac{b_0^2}{1 - 2a_1\cos(\hat{\omega}) + a_1^2}.\end{aligned}$$

(i) $0 < a_1 < 1$: a positive pole at $z = a_1$ and a peak at $\hat{\omega} = 0 \implies$ a lowpass filter

(ii) $-1 < a_1 < 0$: a negative pole at $z = a_1$ and a peak at $\hat{\omega} = \pi \implies$ a highpass filter





2 Second-Order IIR Filter

(a) **Difference equation:** a_1, a_2 and b_0 real

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n].$$

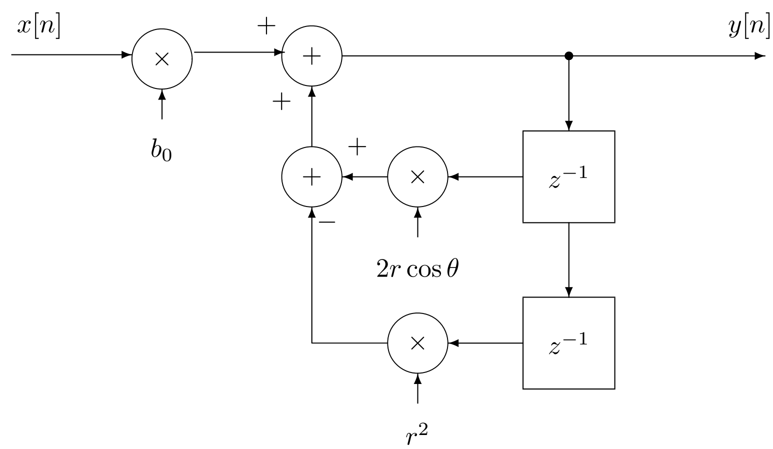
(b) **System function:**

$$\begin{aligned} H(z) &= \frac{b_0}{1 - a_1 z^{-1} - a_2 z^{-2}} \\ &= \frac{b_0 z^2}{z^2 - a_1 z - a_2} \\ &= \frac{b_0 z^2}{(z - re^{j\theta})(z - re^{-j\theta})} \\ &= \frac{b_0}{1 - 2r\cos(\theta) z^{-1} + r^2 z^{-1}}. \end{aligned}$$

(c) **Impulse response:**

$$h[n] = \frac{1}{\sin\theta} r^n \sin\big((n+1)\theta\big) u[n].$$

(d) **Implementation:**



(e) **Stability requirement:** The two poles must reside inside the unit circle $|r| < 1$

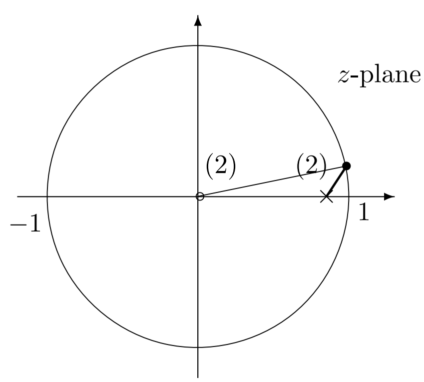
(f) **Frequency response function:**

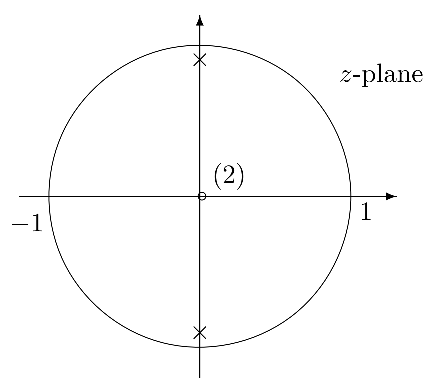
$$\mathcal{H}(\hat{\omega}) = H(z)\Big|_{z=e^{j\hat{\omega}}} .$$

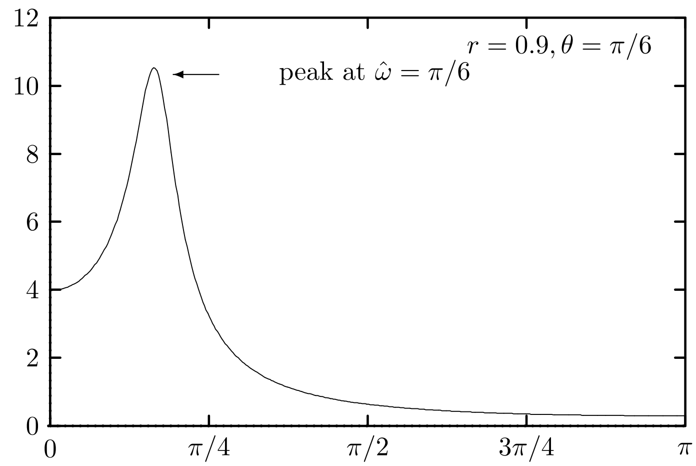
(i) Special cases: $\theta = 0$ or π

Two poles (therefore, a peak) at $z = 0$ or $z = \pi \implies$ a lowpass or highpass filter

(ii) $\theta \neq 0, \pi$: a peak at $\hat{\omega} = \theta \implies$ a bandpass filter.







Conclusion: A real coefficient 2nd order IIR filter can be used as a building block for low, high or bandpass filtering.