• First-order IIR filter
• Second-order IIR filter

1 First-Order IIR Filter

(a) **Difference equation**: $a_1$ and $b_0$ real

$$y[n] = a_1 y[n-1] + b_0 x[n].$$

(b) **System function**:

$$H(z) = \frac{b_0}{1-a_1 z^{-1}} = \frac{b_0 z}{z-a_1}.$$

(c) **Impulse response**:

$$h[n] = b_0 a_1^n u[n].$$

(d) **Implementation**:

![Implementation Diagram]

(e) **Stability requirement**: $|a_1| < 1$

(f) **Frequency response function**:

$$|H(\hat{\omega})|^2 = \left|\frac{b_0}{1 - 2a_1 \cos(\hat{\omega}) + a_1^2}\right|^2.$$

(i) $0 < a_1 < 1$: a positive pole at $z = a_1$ and a peak at $\hat{\omega} = 0 \implies$ a lowpass filter

(ii) $-1 < a_1 < 0$: a negative pole at $z = a_1$ and a peak at $\hat{\omega} = \pi \implies$ a highpass filter
2 Second-Order IIR Filter

(a) Difference equation: $a_1, a_2$ and $b_0$ real

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n].$$

(b) System function:

$$H(z) = \frac{b_0}{1 - a_1 z^{-1} - a_2 z^{-2}} = \frac{b_0 z^2}{z^2 - a_1 z - a_2} = \frac{b_0 z^2}{(z - r e^{j\theta})(z - r e^{-j\theta})} = \frac{b_0}{1 - 2r \cos(\theta) z^{-1} + r^2 z^{-2}}.$$

(c) Impulse response:

$$h[n] = \frac{1}{\sin \theta} r^n \sin ((n+1)\theta) u[n].$$
(d) Implementation:

\[ y[n] = x[n] \times b_0 + 2r \cos \theta \times (z^{-1})^2 \]

(e) **Stability requirement:** The two poles must reside inside the unit circle \(|r| < 1\)

(f) **Frequency response function:**

\[ H(\hat{\omega}) = H(z) \mid_{z=e^{j\hat{\omega}}} \]

(i) Special cases: \( \theta = 0 \) or \( \pi \)

Two poles (therefore, a peak) at \( z = 0 \) or \( z = \pi \) \( \Rightarrow \) a lowpass or highpass filter

(ii) \( \theta \neq 0, \pi \): a peak at \( \hat{\omega} = \theta \) \( \Rightarrow \) a bandpass filter.
Conclusion: A real coefficient 2nd order IIR filter can be used as a building block for low, high or bandpass filtering.