This is a 120 minute exam.

- It is closed book, closed notes, closed computer.
- You may use three 8.5x11” piece of papers, both sides, and a calculator.
- There are 10 problems for a total of 100 points. The questions are not necessarily in order of increasing difficulty.
- This exam has 6 pages. Make sure your copy is complete.
- Continuing to write anything after the ending time is announced will be considered an honor code violation. *Fill out your name etc. above now, and do not wait until the end to circle your answers!*
- Clearly circle your final answers. For full credit, show your complete work clearly on all problems. (For some problems, we may grade only the final answer nevertheless.)
1. (10 points)
A signal $x(t)$ has the following spectrum.

![Spectrum of x(t)](image)

The signal is passed through an ideal anti-alias filter, then sampled with sampling interval $T_s = 1/30$ seconds, then interpolated by the ideal sinc interpolator as follows.

$$x(t) \rightarrow \text{ideal anti-alias filter} \rightarrow \text{sample at } T_s \rightarrow \text{ideal interpolator} \rightarrow y(t)$$

Carefully sketch the spectrum of the output signal $y(t)$.

2. (10 points)
A continuous-time signal $x(t)$ is sampled without aliasing at rate $f_s = 1000$Hz to yield a discrete-time signal $x[n]$. The spectrum of $x[n]$ is the following.

![Spectrum of x[n]](image)

Could this signal have been sampled at an even lower sampling rate without aliasing? If so, describe what sampling rates $f_s$ would prevent aliasing. If not, explain why not.
3. (10 points)

The 8-periodic signal $x[n]$ having 8-point DFT given by $X[k] = \begin{cases} 2, & k = 0 \\ 3, & k = 4 \\ 0, & \text{otherwise} \end{cases}$ is the input to a FIR filter with impulse response $h[n] = \delta[n] + 2\delta[n - 4]$. Determine the output signal.

4. (10 points)

A filter has the following pole-zero plot.

Determine the response of this filter to the input signal $x[n] = 3\cos\left(\frac{\pi}{4}n\right)$.
5. (10 points)
An LTI system has frequency response
\[ \mathcal{H}(\hat{\omega}) = 2e^{-j\hat{\omega}} + 3e^{-j5\hat{\omega}}. \]
Determine the response of this system to the following input signal
\[ x[n] = 7\delta[n + 1] + 9\delta[n - 3] - 6. \]

6. (10 points)
Determine the impulse response of the filter that has the following frequency response
\[ \mathcal{H}(\hat{\omega}) = \frac{e^{-j2\hat{\omega}} - 5e^{-j3\hat{\omega}}}{1 + e^{-j\hat{\omega}}}. \]
7. (10 points)
A filter has the following impulse response

\[ h[n] = (\frac{1}{2})^n u[n] + (\frac{1}{2})^{n-1} u[n - 1]. \]

The (non-ideal) magnitude response of this filter best corresponds to which of the following? Explain.

a) Coffee filter  
     b) Notch filter
     c) Highpass filter  
     d) Lowpass filter 
     e) Bandpass filter
     f) Resonator

8. (10 points)
Determine the difference equation for the filter that has the following impulse response

\[ h[n] = (\frac{1}{2})^n u[n] + (\frac{1}{2})^{n-1} u[n - 1]. \]
9. (10 points)
An continuous-time input signal $x(t)$ is sampled, processed by a discrete-time system, and then interpolated to form an output signal $y(t)$ as follows.

\[ H_1(z) = 1 + \frac{1}{2}z^{-3} \]

\[ h_2[n] = b\delta[n - 1] + \delta[n - 2] \]

Determine the value of the coefficient $b$ for which a 125 Hz sinusoidal input signal will produce an output signal $y(t)$ that is completely zero.

10. (10 points)
A signal processing system consists of two filters and a multiplier $(\otimes)$, connected as follows.

\[ Filter1: h_1[n] = \delta[n] - \delta[n - 1] \]

\[ Filter2: H_2(z) = 1 + z^{-2} \]

Assuming the input signal is $x[n] = \cos(\pi n) + \cos\left(\frac{4}{3}n\right)$, carefully sketch the spectrum of the output signal $y[n]$. Hint. To maximize partial credit, first determine $y_1[n]$ and $y_2[n]$, the outputs of the two filters.