This final exam review is meant to be a comprehensive outline to all the topics covered in the course. Examples are not included. Sample problems that demonstrate some of these concepts are given in a separate paper.

- **Signal Properties**
  - Max, Min, Mean, Energy, Mean Square, RMS
  - Be able to use MSD (just mean square value of the difference) to compare difference (or error) in signals

- **Translation and scaling of a signal**
  - How signal properties are affected by scaling and shifting in amplitude
  - How signal properties are affected by scaling and shifting in time
  - Determine a the results of shifting/scaling in amplitude and time (i.e., if given \( x(t) \), be able to find \( y(t) = a + bx(ct + d) \))
    * Recall that shifting in amplitude affects time outside of the original support region (e.g. See HW2, problem 4, sketch of \( y(t) \))

- **Signal Value Distribution (SVD)**
  - Essentially a histogram
  - Information on time sequence of the signal is lost (so shifting/scaling of time will have no effect on SVD)
  - The vertical axis of the signal becomes the horizontal axis of the SVD (see HW2, problem 2)

- **Periodic signals—sinusoids (estimation of amplitude, frequency, phase)**
  - Relate parameters amplitude, frequency, phase to a figure of the signal.
  - Convert to standard form (e.g., use \( \sin(t) = \cos(t - \pi/2) \) and keep \(-\pi < \Phi \leq \pi\))

- **Periodic signals—complex exponentials**
  - Distinguish between the periodic part versus the phasor
  - Be able to convert to cosine using just the real part

- **Complex arithmetic**
  - Euler and inverse
  - Have some specific angles memorized (like \( e^{j\pi/2} = j, e^{j\pi} = -1, e^{-j\pi/2} = e^{j3\pi/2} = -j \))
  - Convert between polar (magnitude and phase) and Cartesian form of complex number
  - Know things like \( |z|^2 = zz^* \) and similar for division
  - Note that phasors are just complex numbers and should be added as such (see section on adding sinusoids)

- **Operations on two signals—adding sinusoids of same frequency**
  - Convert sinusoids to real part of complex exponential
  - Separate the periodic part from the phasor
  - Deal with phasors as complex numbers

- **Adding periodic signals - periodicity of the result**
  - If ratio of periods of each periodic signal is rational number \( \Rightarrow \) periodic
  - If ratio of periods of each periodic signal is irrational number \( \Rightarrow \) not periodic

- **Correlation and normalized correlation is analogous to vector dot product (it indicates how much a signal is in the “direction” of another signal)**

- **Spectral Representation of Continuous Time Signals (infinite duration)**
  - Indicates that the signal is the sum of sinusoids
– Magnitude is symmetric about origin while phase is conjugate symmetric
– Periodicity of the sum depends upon same rules as adding periodic signals (above)
– Spectra and Time Scaling/Shifting
  * If given spectrum of \( x(t) \) and \( y(t) = a + bx(ct + d) \), be able to calculate spectrum of \( y(t) \) (page 3a.10) (Linearity properties).
  * Be able to add/subtract spectra of two signals (addition of two spectral lines at same frequency is just phasor addition)
– Fourier Series
  * A specific type of spectra with frequency components represented as a fundamental and multiples of it
  * Have the synthesis and analysis formulas handy (pp. 3a.14, 3a.15)
  * Should be able to calculate frequency from k-value (\( f_k = f_0 \cdot k \) or \( f_k = k/T_0 \))
  * Parseval’s theorem
  * Know (or have written) most of properties (linearity, time shifting, frequency shifting, etc.)

• Spectral Representation of Discrete Time Signals (infinite duration)
– Discrete-time signal may not be periodic even though it has a single spectral line
  * Frequency (\( \hat{\omega} \) in radian/sample) must be rational number times \( \pi \) in order to be periodic
– If each spectral line in discrete-time spectrum represents periodic signal, then the sum is periodic
– Be able to convert easily between 1-sided and 2-sided spectrum
– DFT - analogous to Fourier Series
  * know the analysis and synthesis equations, Parseval’s theorem
  * Know (or have written) most of properties (linearity, time shifting, frequency shifting, etc.)
    · Be able to relate \( k \) to \( \hat{\omega} \) using \( \hat{\omega} = \frac{2\pi k}{N} \)
    · Recognize \( -k \) is equivalent to \( N - k \)
    · Combine equivalent frequencies (\( \hat{\omega} \) and \( \hat{\omega} + 2\pi n \) are equivalent when \( n \in \mathbb{Z} \))
  * Important to note—it is NOT infinite
  * Be able to use coefficient matching to quickly do synthesis/analysis

• Sampling
– Understand \( x[n] = x(nT_s) \)
– Convert to/from sampled frequency using \( \hat{\omega} = \omega/f_s \)
– Be able to find aliased frequencies (frequencies in the discrete world live on a circle)
  * Case 1 aliasing - aliased but not folded (phase matches that of continuous time signal)
  * Case 2 aliasing - aliased and folded (phase has opposite sign as that of continuous time signal)
– Given a quantization range and number of quantization levels, determine what the levels are (HW 13, problem 7 and Lab 5)
– Quantization error and how it affects spectrum (HW 13, problem 7)
– Sampling at the critical frequency (\( \hat{\omega} = \pi \) exactly) \( \Leftrightarrow \) aliasing occurs (since we can cannot determine both phase and magnitude)
– Anti-aliasing filter (a continuous-time filter that acts before sampling occurs) will remove all signals at or above sampled frequency of \( \pi \) (note: cannot realize a perfect anti-aliasing filter!)

• Reconstruction of Sampled Signals
– Zero-order hold (a pulse drawn at the sample)
– First-order (linear) interpolator (connect the dots)
– Ideal (sinc) interpolator (uses all past and present data and can theoretically reconstruct original if not aliased)
If ideal interpolator assumed, be able to convert between sampled signal and corresponding continuous
time signal by inspection.

• Linear, time-invariant systems (LTI)
  
  • Linear defined by possessing properties of superposition and homogeneity
    
    - Superposition - adding two input signals is equivalent to inputting each separately then adding the
      outputs
    
    - Homogeneity - scaling an input signal is equivalent to inputting the signal, then scaling the output
  
  • Time-invariance requires that shifting an input signal be equivalent to shifting the output
  
  • LTI properties can be used to calculate output in response to an input by characterizing response to
    scaled, shifted, or superposed versions of it (see HW 10, problem 9b)

• Causal systems
  
  • Require that output be a function of input at only current and past time
  
  • Not affected by signals unrelated to input (e.g., constant output or sinusoid output that is not due to
    input)

• Filtering using Finite Impulse Response (FIR)
  
  • LTl system
  
  • Can summarize all we need to know in terms of \( h[n] \), impulse response
  
  • Can represent as difference equation of the form
    \[
    y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3] + \cdots
    \]
  
  • Impulse response is (almost trivially) obtained by substituting \( \delta[n] \) for \( x[n] \)
  
  • Convolution used to calculate time domain response to finite-length input signals
  
  • Simple algorithm (like 3rd grade multiplication) works for this case (p. 132 of textbook) using coefficients
    of \( h[n] \) and of \( x[n] \)

• Filtering using Infinite Impulse Response (IIR)
  
  • LTl system (like FIR)
  
  • Can summarize all we need to know in terms of \( h[n] \), impulse response (like FIR)
  
  • Can represent as difference equation of the form
    \[
    y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3] + \cdots + a_1 y[n-1] + a_2 y[n-2] + a_3 y[n-3] + \cdots
    \]
  
  • Impulse response is infinite so contains terms like \( a^n u[n] \)
  
  • Convolution still applies, but is impossible to work out to completion

• z-transform
  
  • Simplifies calculations by allowing FIR/IIR filters to be expressed as ratio of polynomials
  
  • Frequency response is intimately related to location of roots of polynomials
    
    - Roots of numerator (FIR part) are zeros of system
    
    - Roots of denominator (IIR part) are poles of system
    
    - Roots of polynomials are found by factoring, just as in high school algebra class
  
  • Allows us to express convolution as multiplication of polynomials (which is all it is anyway … compare
    page 132 to page 216 of textbook)
  
  • Calculation of z-transform from difference equation (almost trivial)
    
    - \( y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \cdots + a_1 y[n-1] + a_2 y[n-2] + \cdots \)
    
    - \( Y(z) = b_0 X(z) + b_1 X(z)z^{-1} + b_2 X(z)z^{-2} + \cdots + a_1 Y(z)z^{-1} + a_2 Y(z)z^{-2} + \cdots \)
  
  • Cascade of systems is product of z-transforms (product of ratios of polynomials)
  
  • Be able to use partial fraction expansion (see HW 13)
Final Exam Review

– NOTE: WE DO NOT YET KNOW HOW TO TAKE z-TRANSFORM OF INFINITE SUPPORT SIGNALS, only of systems and finite support signals
  * The text computes transfer function in z-domain by ASSUMING the z-transforms of the input and output signals exist
  * Take EECS 306 to learn more about z-transforms of infinite support signals

• Frequency Response
  – Just a complex number with the magnitude and phase functions of \( \hat{\omega} \)
  – Can simplify only for “simple filters”
    * When symmetric, can pull out the middle term (Example 6.1 of textbook)
    * In this case, phase is linear with frequency (times half the order of the filter)
  – Magnitude is equivalent to the “height” of \( H(z) \) (system function) evaluated on the unit circle (good visualization is Figure 8.23 of textbook)
  – Equivalently, \( H(\hat{\omega}) = H(z)|_{z = e^{j\hat{\omega}}} \)
  – Magnitude is always a nonnegative number
  – Phase changes in jumps when
    * Phase must change by \( \pi \) to keep magnitude positive
    * Phase must remain between \( -\pi \) and \( \pi \) (reducing \( \theta \) modulo \( 2\pi \))
  – Might be easier to calculate at a few frequencies if all that is needed (rather than try to simplify to a general magnitude and phase form)
  – Can visualize frequency response magnitude at a point on the unit circle as product of distances from the point to zeros, divided by product of distances from same point to poles (Figure 8.24 of textbook)

• Special Filters
  – N-point moving average
    * Frequency Response - magnitude is Dirichlet function phase is linear (pp. 180,181 of textbook)
    * z-transform has N-1 zeros on unit circle (p. 228 ff of textbook)
  – Nulling filters (designed to place a null or zero at a particular frequency)
    * Filters with real coefficients will null complex conjugate frequencies
    * The set of frequencies nulled by a cascade of nulling filters is the union of the frequencies nulled by each filter
  – Bandpass filters
    * Analogous to a pair of N-point moving average filters “rotated” in the z-plane from \( \hat{\omega} = 0 \) to another pair of complex conjugate frequencies
    * z-transform has N-2 zeros on the unit circle and another zero inside the unit circle

• Miscellany
  – Signal “suddenly applied” is just product of infinite support signal with unit step function
    * For FIR, results in transient response up to order of filter in length, steady state thereafter
    * For IIR, transient response decreases exponentially but never goes away (in theory)
  – Should be able to sketch frequency response magnitude from pole/zero plot
  – Should be able to sketch impulse response from pole/zero plot
  – If given input to filter and asked to find output
    * Use \( h[n] \) and convolution (or equivalently, \( H(z) \)) to find output due to those components of input that are of the form \( c_0\delta[n] + c_1\delta[n - 1] + c_2\delta[n - 2] \ldots c_N\delta[n - N] \)
    * Use frequency response \( H(\hat{\omega}) \) to find output (steady state) due to those components of input that are of the form \( a\cos(\omega n + \phi) \)
    * DC signals treated as signals at \( \hat{\omega} = 0 \)
    * If input “suddenly applied,” then use steady state after transient is gone, but calculate transient using \( h[n] \) and convolution.
    * LTI allows summing outputs calculated by all methods