Notes

- Review the policies on HW1! (Except, now you turn in HW in 2420 EECS, which is open 24/7.)
- Reading: all of “Part 1” lecture notes.

Skill Problems

1. [50] Concept(s): signal characteristics: energy, mean, average power, RMS value
   For each of the following signals, determine the energy [3], the mean value [3], the average power [4], and the RMS value [0]. Hints: sketching the signals will help, and look at the “useful formulas” page.
   (a) \[ x_1(t) = \begin{cases} \cos\left(2\pi \frac{1}{T_0} t\right), & t \geq 0 \\ 0, & \text{otherwise}. \end{cases} \]
   Answer: \( E(x_1) = \infty, M(x_1) = 0, MS(x_1) = 1/2, RMS(x_1) = 1/\sqrt{2} \)
   (b) \[ x_2(t) = \begin{cases} 1, & 0 < t < 1 \\ 1 - |t - 5|, & |t - 5| \leq 1 \\ 0, & \text{otherwise}. \end{cases} \]
   (c) \[ x_3(t) = \begin{cases} \cos^2\left(2\pi \frac{1}{T_0} t\right), & 0 \leq t \leq T_0 \\ 0, & \text{otherwise}. \]
   Hint: look for \( \cos^2 \) in the list of useful formulas.
   Hint: RMS(\( x_3 \)) = \( \sqrt{3}/8 \). (But do not use this to solve for other values.)
   (d) \[ x_4(t) = 7 + 3e^{-2|t|} \]
   (e) \[ y_1[n] = \cos\left(\frac{2\pi}{4} n\right) \]
   (f) \[ y_2[n] = \begin{cases} 1 + (1/3)^n, & n \geq 0, \\ 0, & \text{otherwise} \end{cases} \]

2. [20] Concept(s): signal value distributions / histograms
   (a) [10] Determine the signal value distribution of signal \( x[n] = \sin\left(\frac{2\pi}{4} n\right) \).
   Hint: use one period and sketch the limit as the number of histogram bins increases.
   (b) [10] A discrete-time signal \( x[n] \) has the following histogram.

   ![Histogram](image)

   Determine the mean value \( M(x) \) and standard deviation \( \sigma(x) \) of this signal.
   Hint: see if your values make sense in light of the center and “spread” of the histogram.
   (c) [0] Determine the signal value distribution of signal \( x(t) = \begin{cases} 3, & |t| \leq 1 \\ -2, & 5 < |t| \leq 7. \end{cases} \)
   (Think about what happens as the number of signal samples increases, and express your singular value distribution using proportions.)

3. [20] Concept(s): periodicity, fundamental period, least common multiple
   Determine whether each of the following signals is periodic, and if so, determine its fundamental period.
   (a) [0] \( x_1(t) = \cos(\sqrt{2}\pi t) \). Answer: periodic with fundamental period \( T_1 = \sqrt{2} \).
   (b) [5] \( x_2(t) = \cos(2\pi 200t) + \cos(2\pi 500t) + \cos(2\pi 50t) \)
   (c) [5] \( x_3(t) = \cos(2\pi 200t) + \cos(500t) \)
   (d) [0] \( y_1[n] = \cos\left(\frac{\pi}{4} n\right) \). Answer: periodic with fundamental period \( T_1 = 10 \).
   (e) [0] \( y_2[n] = \cos(7\pi n) \). Hint: sketch it! (The period is not \( 2/7 \).)
   (f) [5] \( y_3[n] = \cos(\sqrt{2}\pi n) \)
   (g) [5] \( y_4[n] = y_1[n] + y_2[n] \), where \( y_1[n] \) and \( y_2[n] \) are as defined in preceding parts.
4. [10] Concept(s): **signal operations: amplitude shift/scale and time shift/scale**

Consider the following signal: 
\[ x(t) = \begin{cases} 
1 - \frac{t}{3}, & \text{if } |t| < 3 \\
0, & \text{otherwise} 
\end{cases} \]

(a) [5] Sketch \( y(t) = 3 + 7x(t/2) \) carefully.

(b) [5] Sketch \( z(t) = x(\frac{2-t}{3}) \) carefully.

5. [45] Concept(s): **effects of signal operations on signal characteristics**

Make a table like the following and complete it. For each entry in the table, write Y if the operation could affect the characteristic or N if it cannot. (The first column is completed for you.) If you are unsure, write “U” instead. For this problem, only your table will be graded, not your work.

Scoring: 3 points for each correct answer, 0 points for wrong answer, except 1 point for each U.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Operation</th>
<th>min/max</th>
<th>duration</th>
<th>energy</th>
<th>mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time shift</td>
<td>N</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time scale</td>
<td>N</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time reversal</td>
<td>N</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amplitude shift</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amplitude scale</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. [20] Concept(s): **Using MATLAB**

Download the MATLAB file `periodic.mat`, using the link on the class website just below the link to this homework set. This file contains a vector called `signal1` that is a segment of a nearly periodic signal.

(a) [0] Start MATLAB, then load `periodic.mat` by typing `load periodic` at the MATLAB prompt.

   (You will need to understand MATLAB’s “path” settings for this to work.)

(b) [5] Determine the length of the vector `signal1`.

(c) [5] Determine its maximum and minimum values.

(d) [10] Determine, approximately, the period of this approximately periodic discrete-time signal. Explain briefly how you determined the value.

For each part, show the MATLAB command(s) that you used. (Handwritten is fine.)

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**Mastery Problems**

7. [25] (a) [10] Determine the mean value and average power of the signal \( x_1(t) = a + b \cos(2\pi ft) \).

(b) [5] Find a signal \( x_2(t) \) that has \( M(x) = 5 \) and \( MS(x) = 50 \).

(c) [10] Find a signal \( x(t) \) that has duration = 5, \( E(x) = 45 \), and \( M(x) = 3 \).

8. [10] Concept(s): **combining time-scaling and periodicity**

Let \( x(t) \) be the following continuous-time periodic signal.

![Periodic Signal](image)

Determine a period \( T \) of the signal \( y(t) = x(t) + x(2t/3 - 1) + x(7t) \).

(Determining whether your answer is the fundamental period is optional.)

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**Optional Extra Credit Problems**

9. [10] An engineer working on a digital satellite communications system was asked by her boss to find a signal \( x(t) \) that has mean value = 5 and average power = 20.

Should her boss take a refresher course in signal characteristics? Explain.