Notes

- Review the HW policies on HW1!
- Reading: Ch. 2 of text and Appendix A of text (complex numbers).
- Exam 1 on Feb. 6 will cover the material corresponding to HW1 through HW4, *i.e.*, the Part 1 lecture notes and text Ch. 2 and Appendix A.
- For exam practice, try the 206 exams from previous semesters, all of which are on the web site.
- For exam review, go any lab section Feb. 4-6.
- This HW will graded and returned after Exam 1. You may want to photocopy your answers before handing them in so you can compare them to the solutions.

Skill Problems

1. [15] Concept(s): **correlation and the effect of signal operations**
   
   In class it was stated that $C_N(ax, by) = C_N(x, y)$. This is correct if $ab > 0$, but not quite if $ab < 0$. Let $x(t), y(t),$ and $z(t)$ denote signals, and let $a$ and $b$ denote nonzero real numbers.
   
   Show the following relationships.
   
   (a) [5] $C(ax, by) = abC(x, y)$
   
   (b) [5] $C(x, y + z) = C(x, y) + C(x, z)$. (These first two properties are called **bilinearity**.)
   
   (c) [0] $C(x, y) = C(y, x)$ (for real signals)
   
   (d) [5] $C_N(ax, by) = \begin{cases} C_N(x, y), & ab > 0 \\ -C_N(x, y), & ab < 0. \end{cases}$
   
   [0] What happens if $ab = 0$?
   
   (e) [0] $C_N(x, \alpha x) = \begin{cases} 1, & \alpha > 0 \\ -1, & \alpha < 0. \end{cases}$ (In fact, $C_N(x, y) = \pm 1$ if and only if $y$ is an amplitude-scaled version of $x$, so $y$ and $x$ have identical “shapes.”)
   
   (Think about how correlation is affected by other signal operations, e.g., amplitude shift.)

2. [25] Concept(s): **representations of sinusoidal signals**

   When relevant below, use the principal value for the phase: $-\pi < \phi \leq \pi$.
   
   (a) [5] A sinusoidal signal $x(t)$ has amplitude=5, frequency 40Hz, and phase = $\pi/3$ radians.
   
   Sketch $x(t)$ carefully by hand, labeling your axes.
   
   (b) [5] Express the following signal in the standard form, *i.e.*, in the form $A \cos(2\pi f_0 t + \phi)$:
   
   $$y(t) = -7 \sin(8\pi(t - 3) + 13\pi/4).$$
   
   (c) [5] Find an expression in standard form for the following sinusoidal signal.
   
   (d) [5] Simplify the following sum of sinusoidal signals into standard form:
   
   $$s(t) = 5 \sin(8t) + 5 \cos(8t - \pi/3).$$
   
   (e) [5] Find a complex-valued signal $\bar{x}(t)$ such that $x(t) = \text{Re}(\bar{x}(t))$, for $x(t)$ as defined in part (a).
3. [30] Concept(s): complex arithmetic
   (a) [15] Convert the following complex numbers from cartesian form to complex exponential form and plot in the complex plane: 
   \( z_1 = \sqrt{3} + j \), \( z_2 = -\sqrt{2} + j \sqrt{2} \), \( z_3 = -2 - j \).
   (b) [10] Determine the product of \( z_1 \) and \( z_2 \) by:
   - performing multiplication entirely in cartesian coordinates,
   - performing multiplication entirely with the exponential forms of these complex variables.
   (c) [10] Determine the ratio \( z_1 / z_2 \) by:
   - performing division by first converting \( z_1 \) and \( z_2 \) to exponential form,
   - performing division by multiplying the numerator and denominator of \( z_1 / z_2 \) by \( z_2^* \).
   (d) [0] Which form is easier for multiplication and division? What about for addition and subtraction?

4. [30] Concept(s): complex arithmetic
   Simplify the following complex-valued expressions.
   For (a)-(d), give answers in both Cartesian form and exponential (or polar) form.
   (a) [4] \( 2e^{j\pi/3} + 4e^{-j\pi/6} \)
   (b) [4] \((\sqrt{3} - j 3)^9 \)
   (c) [4] \((\sqrt{3} - j 3)^{-1} \)
   (d) [4] \((\sqrt{3} - j 3)^{1/3} \). Hint: there are three answers.
   (e) [4] \( \text{Re}(j e^{-j\pi/3}) \)
   (f) [10] Determine all solutions \( \theta \) (in radians) to the following equation: \( \text{Re}(1 - j) e^{j\theta}) = -1 \).

5. [15] Concept(s): sums of sinusoidal signals with same frequency and phasors, effect of time shift/scale
   Simplify the following sums of sinusoidal signals into standard form.
   (a) [0] \( x_1(t) = 5 \cos(2t + \pi/4) + 5 \cos(2t + 3\pi/4) - \cos(2t + \pi/2) \).
      Hint. Using phasors, \( x_1(t) = (5\sqrt{2} - 1) \cos(2t + \pi/2) \approx 6.071 \cos(2t + \pi/2) \).
   (b) [10] \( x_2(t) = 5 \cos(\pi t + \pi/2) + 5 \sin(\pi t - \pi/6) - \cos(\pi t - 2\pi/3) \)
   (c) [5] \( x_3(t) = x_1(-3(t - 2)) \), where \( x_1(t) \) was defined in (a).

Mastery Problems

6. [20] Concept(s): sinusoids and linear systems
   Sinusoidal signals are particularly important because when a sinusoid is the input to a linear time-invariance (LTI) system, the output is also a sinusoid, and this property is unique to sinusoids!
   Consider a system with an "echo:" the output signal \( y(t) \) is the sum of the input signal \( x(t) \) and a delayed version of \( x(t) \). (You may have experienced something like this in some cell phone calls.) Assume that the following input/output relationship describes the system: \( y(t) = x(t) + x(t - 1) \).
   (a) [10] If \( x(t) = A \cos(2\pi f_1 t) \), show that the output \( y(t) \) can be written as \( B \cos(2\pi f_2 t + \phi) \).
      Relate \( B, \phi \) and \( f_2 \) to \( A \) and \( f_1 \). This is called the "sine in, sine out" property.
      Hint. Use this "phase splitting" trick: \( 1 + e^{j\gamma} = e^{-j\gamma/2} [e^{j\gamma/2} + e^{-j\gamma/2}] = e^{-j\gamma/2} 2 \cos(\gamma/2) \).
   (b) [10] Now instead of a sinusoidal input, suppose that the input signal \( x(t) \) is periodic with period 4 and \( x(t) = 1 \) for \( 0 < t < 2 \) and \( x(t) = 0 \) for \( 2 < t < 4 \). Sketch the input \( x(t) \) and the output signal \( y(t) \).
      [0] Is there a "square wave in, square wave out" property?

7. [15] Concept(s): Euler’s formula
   (a) [10] Prove the following equality, called DeMoivre’s formula, using Euler’s formula.
      \[ (\cos \theta + j \sin \theta)^n = \cos(n\theta) + j \sin(n\theta). \]
   (b) [5] Use this result to evaluate \( (3 + j 4)^{99} \).