Notes

- Review the HW policies on HW1!
- Final exam information is on HW11. We will check UM ID’s at the final exam.
- Reading: Text Ch. 7.
- Relevant practice problems on the DSP CDROM: 7.3, 7.4, 7.18, 7.21, 7.25, 7.32, 7.35, 7.46, 7.49

Skill Problems

1. [0] Text 7.1. Concept(s): *(z-transform of finite sequences)*

   Hint: \( X_4(z) = 2 - 3z^{-1} + 4z^{-3} \)

2. [0] Text 7.3. Concept(s): *(diffeq and response from system function)*

   Hint: \( h[n] = \delta[n] + 5\delta[n - 1] - 3\delta[n - 2] + 2.5\delta[n - 3] + 4\delta[n - 8] \)

3. [30] Text 7.8. Concept(s): *(diffeq to \( h[n] \), \( H(z) \), zplane, \( H(\omega) \))*

4. [15] Text 7.9ace. Concept(s): *(Cascade of two \( H(z) \)'s)*

5. [15] Concept(s): *system function and frequency response*

   A filter has input-output relation

   \[ y[n] = x[n - 1] + \sqrt{2}x[n - 3] + x[n - 5]. \]

   (a) [5] By taking the Z-transform of both sides of this equation to obtain \( Y(z) = H(z)X(z) \), identify the system function \( H(z) \).

   (b) [5] From the system function of part (a), show that there is a frequency \( \omega_0 \) for which the output signal is zero when \( x[n] = A \cos(\omega_0 n + \phi) \) for any \( A, \phi \). What is this frequency?

   (c) [5] Determine the output signal \( y[n] \) when the input is the above sinusoid with \( A = 10, \omega_0 = \pi/5 \) and \( \phi = 0 \).
Mastery Problems

6. [25] Concept(s): suddenly applied signals

The input to an LTI system is the following discrete-time signal, which is periodic for \( n \geq 0 \).

The frequency response of the system is given by

\[
H(\omega) = \frac{1}{4} \left( 1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} \right).
\]

(a) [20] Determine a formula for the output signal \( y[n] \) without using braces.
(b) [5] Sketch the output signal \( y[n] \), and identify which part is the transient response.

7. [30] Concept(s): filtering of sampled continuous-time signals

The above sawtooth signal \( x(t) \) is the input to the following connected systems:

\[
\begin{aligned}
x(t) &\rightarrow \text{Ideal anti-alias filter} \rightarrow x_a(t) \rightarrow \text{Ideal C-D} \rightarrow x[n] \rightarrow \text{LTI} \rightarrow y[n] \rightarrow \text{Ideal D-C} \rightarrow y(t).
\end{aligned}
\]

(a) [0] Sketch the spectrum of \( x(t) \). (See HW6.)
(b) [5] Sketch the spectrum of \( x_a(t) \).
(c) [10] Sketch the spectrum of \( x[n] \).
(d) [10] Sketch the spectrum of \( y[n] \), assuming that \( h[n] = \frac{1}{2}(\delta[n] + \delta[n-1]) \).
(e) [5] Determine \( y(t) \).

Optional challenge. Design a filter (specify \( h[n] \)) that would remove all of the harmonics except the fundamental frequency for the above sampled sawtooth signal.

8. [10] Concept(s): response to mixed periodic / aperiodic input given \( H(z) \)

Consider the following cascade of LTI systems

\[
\begin{aligned}
x[n] &\rightarrow H_1(z) = 1 + z^{-2} \rightarrow H_2(z) = 1 - z^{-2} \rightarrow y[n].
\end{aligned}
\]

The input to this cascade system is the signal

\[
x[n] = 20 - 7\delta[n - 5] + 20 \cos\left(\frac{\pi}{4}n\right).
\]

Determine the output signal \( y[n] \). Given an equation (no braces!) valid for all \( n \).

Hint. Do not use convolution. Use more than one method and linearity.

9. [0] Will you need your UM ID at the final exam?