Summary of Filter Input-Output Relationships

input signal description	filter description	input-output relationship
arbitrary signal x[n] sequence of samples formula sum of impulses	impulse response sequence h[n] (FIR,IIR)	convolution: $y[n] = x[n]*h[n]$
	FIR filter coef's $\{b_k\}$	FIR difference equation
arbitrary suddenly applied signal x[n]	IIR filter coef's $\{a_k\}$, $\{b_k\}$	IIR difference equation with initial rest condition
complex exponential $e^{j\hat{\omega}n}$		$e^{j\hat{\omega}n} \to H(\hat{\omega})\, e^{j\hat{\omega}n}$
sinusoid $\cos(\hat{\omega}n+\phi)$	frequency response function $H(\hat{\omega})$ (FIR,IIR)	$\cos(\hat{\omega}n+\phi) \to$ $\lvert H(\hat{\omega})\rvert$ $\text{os}(\hat{\omega}n+\phi+\arg(H(\hat{\omega})))$
sum of expon'ls & sinusoids		sum $\to$ sum
periodic signal N point DFT: X[k] (sum of harmonic exponentials)		periodic in $\to$ periodic out $Y[k]=X[k]H(\frac{2\pi}{N})$
suddenly applied sinusoidal signal	FIR: h[n], $H(\hat{\omega})$	hybrid approach: convol'n for n=0 to M-1, FRF for $n \geq M$
abitrary suddenly applied signal x[n], described by z-transform X(z)	system function H(z) (FIR,IIR)	$Y(z) = X(z)\, H(z)$
	poles, zeros and gain of H(z) (FIR,IIR)	indicates: $\lvert H(\hat{\omega})\rvert$

Notes:

1. IIR filters need to be stable in order that
   - frequency response exists
   - convolution of impulse response with any input signal x[n] with support $-\infty$ to ∞ is well defined and finite
   - the response to a sinusoidal input with support $-\infty$ to ∞ is a sinusoid.
   - the response to a suddenly applied sinusoidal input is a sinusoid plus a decaying transient signal
2. an IIR filter is stable if all poles are located inside the unit circle,

   or equivalently, if $\sum_{n=0}^{\infty} |h[n]| < \infty$ (we have not discussed this condition)